Application of the Hough Transform to Parameter Estimation and Control

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Abstract

This paper extended the application of the Hough transform, popular in image processing for line and shape detection, to system parameter estimation and for adaptive control applications. Initially, by way of a proof of concept, the Hough transform is applied to parameter determination of first and second order systems during a standard step response. The approach is then extended to determine system parameters for more general system inputs. An example of directly determining suitable control parameters is presented. This shows that the approach can be incorporated into an adaptive control architecture and to assist in determination of a systems state of health.

Keywords: Data driven control, Pattern recognition control system, Hough Transform

Introduction

Patten recognition methods for control have been around since at least the 1960s [Widrow 1964a, Widrow 1964b]. Patten recognition systems has also been seen as a basis for intelligent control (Ceangă 2004) and using neural networks for the pattern recognition (Wang 2018). Seem (Seem 1998) developed a controller to adapt the proportional and integral gains of first-order plus dead-time systems and applied it to the control of an HVAC system.

Various attempts were been investigated that applied the Hough transform to parameter identification and control. For example, Henderson (Henderson 1997) successfully applied the Hough transform to the parameter identification of linear and nonlinear systems. The approach showed good robustness to noise in comparison to least squares based estimators and suitability to real-time control. Turan (Turan 2003, Turan 2005) also has applied the Hough transform to parameter estimation

Many graphic techniques exist for control systems analysis and design (To 2016), such as the Root locus, Nyquist diagrams and Bode plots. Some of these look suitable to exploitation by pattern recognition systems during control system design phase. For example, Hsu (Hsu 1993) applied the Hough Transform to Lead-lag compensator design using the compensator frequency response.

Xiong (Xiong 2013) has applied a version of the Hough transform at the core of an eye gaze tracking control system for limb disabled people with healthy eyes.

This paper is organised as follows. Section 1 contains a brief overview of the Hough transform and its application to line detection. Section 2 applies the Hough Transform to parameter identification from a systems step response data for first and second order systems. Section 3 extends this approach to parameter identification with arbitrary inputs to the system.

Overview of the Hough Transform

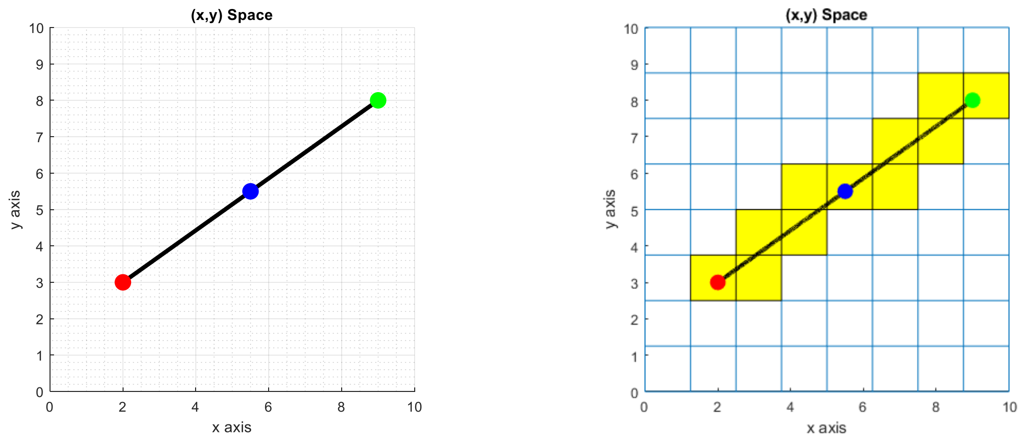
The Hough transform was introduced by Paul Hough in 1965 (Hough 1965) as a method of extricating data from noisy images. It has been applied to wide range of applications from line detection (its original application) to curve following and target tracking (Duda & Hart 1972). It has proved to be a useful technique that copes well with problems of noise and occlusion (Shapiro 2001). The transform provides a mapping from image space to parameter space and converts a difficult global pattern recognition problem into one of local peak detection. The next section, by way of example, applies the Hough transform applied to line detection.

Application to Straight Line Detection

In this section, the problem of straight-line detection using the Hough transform is illustrated. One of the simplest example of the Hough transform is in line detection. Consider a straight line in image space (x, y) given by the equation,

$$y = mx + c \tag{1}$$

Figure 1(a) shows the original line and in (b) the line in an 8x8 course pixelated image. The yellow pixels are the pixel that the line goes through and in a black and white image, after edge detection would be active.



(a) *(b)*

Figure 1. Line in (x,y) space (a) original line (b) line in the quantized (pixelated) image

As can be seen, the pixilation process has degraded the quality of the line. Reordering the equation of the straight-line give the equation of a straight line in parameter space (m, c)

$$c = xm - y \tag{2}$$

Each (active) pixel maps to a line in parameter space. In the example, the red pixel creates the red line in parameter space in Figure 2. The lines in parameter space cross at the coordinates of the line in image space. An accumulator that counts the number of can detect this.

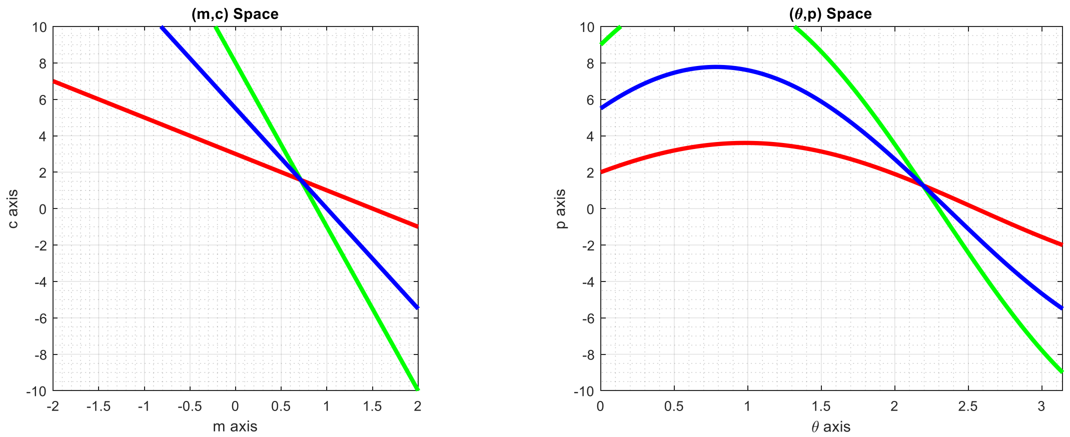


Figure 2. (a) Lines in (m, c) parameter space and (b) in parameter (θ, p) space

To overcome the problem with vertical lines in image space the Hesse normal form can be used [Duda 1972], which has the form

$$r = x\cos(\theta) + y\sin(\theta) \tag{3}$$

This equation gives the curves shown in Figure 2 (b).

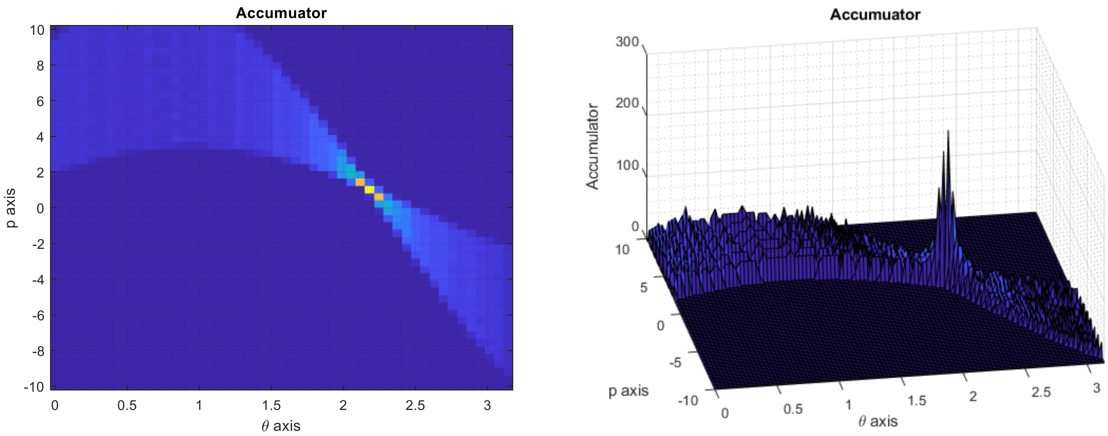


Figure 3. An accumulator in a finely quantized parameter (θ, p) parameter space. The peak is the coordinates of the line in the image.

For an n by m image there are a maximum of nm line parameters that can be isolated. The line parameters are obtained by looking for a peak of an accumulator in parameter space. The approach has proved valuable for the detection of shapes with a low number of parameters and is quite robust to noise.

In applying this concept to the development of a controller, there are many possibilities to use that are equivalent to image space. The system time response to specific inputs, a systems frequency response, the phase plane or a systems input-output time series. The next section will consider the use of a standard step response to obtain system parameters and controller solutions. Here the equivalent to image pixilation is the systems discrete-time sampling rate and the sensor measurement resolution and quantization caused by analog-to-digital converters.

Proof of Concept

A system step response is used to help develop confidence in the approach of applying the Hough Transform to parameter identification, as an initial proof of the concept and to determine which types of systems where it could be applied. Results below show the evaluation of the approach on various systems with relatively few parameters, namely first and second order systems.

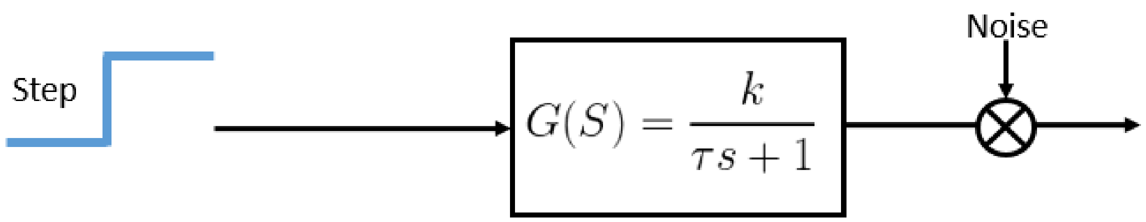


Figure 4. First order system.

Consider a standard system function, a unit step response, applied to a first-order continuous-time linear system, as shown in figure 4. The dynamics of the system are given by equation 4,

$$G(S) = \frac{k}{\tau s + 1} \tag{4}$$

The output time response for a unit step input to such a system is given by equation 5

$$y(t) = k\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right) \tag{5}$$

Every sample data point from the systems step response can then be mapped to a line in (k, τ) parameter space, rearranging equation 5 gives

$$k = \frac{y(t)}{\left(1 - e^{\left(-\frac{t}{\tau}\right)}\right)} \tag{6}$$

An example for a system where $k = 2,\ \tau = 0.7$ is shown in figure 5, where two points on the step response curve have been mapped to lines in (k, τ). These lines cross at the true system parameters.

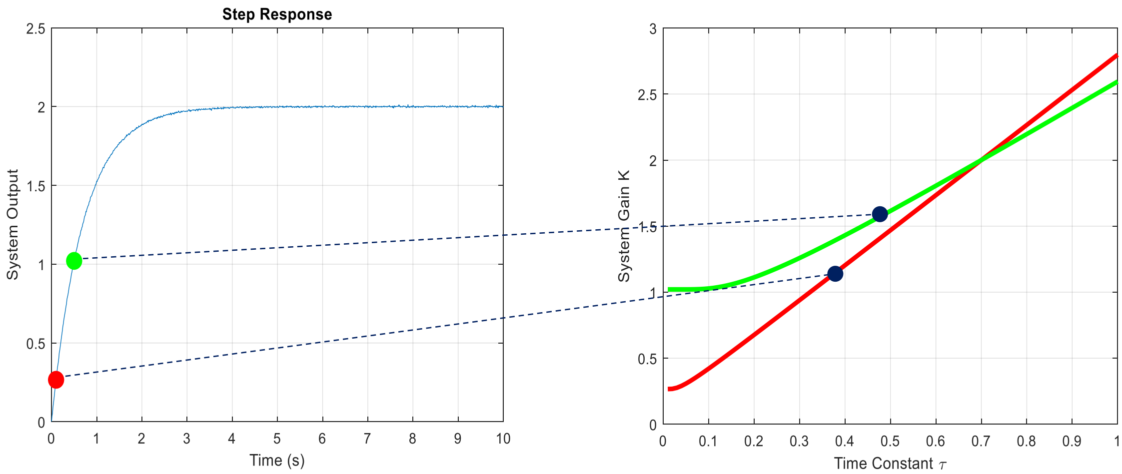
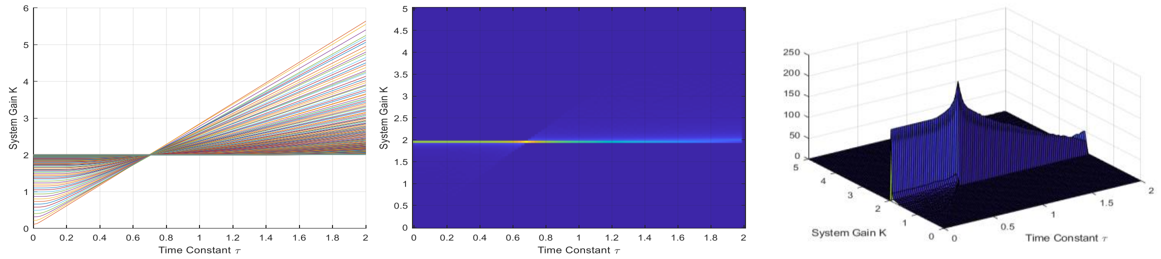


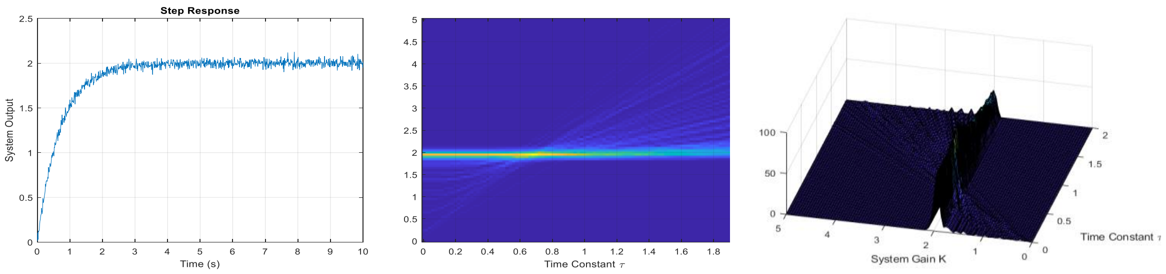
Figure 5. Step response of a first order system.

Figure 6 extends this analysis for more values from the system step response. Figure 6(a) shows the mapping to points on the curve to lines within the (k, τ) parameter space. Figure 6(a) shows a heat map for a quantization of this space where the colour represents the number of values in each region. Figure 6(c) shows the accumulator of lines in the quantized parameter space in 3D where the estimated parameters can be obtained by finding the peak value.

The first three plots of Figure 6 are for the noise free case. The system response can be affected by system and measurement noise. Figure 6 (d-f) show an example for the system response corrupted by noise and the effect on the transform to parameter space. The accumulator space is not as clear with the values distributed more within the parameter space but the system peak is still consistent.



(a) *(b)* *(c)*



(d) *(e)* *(f)*

Figure 6. Parameter estimation for a first order system. (a)-(c) System parameter space for the step response in figure 4 and the accumulator (d)-(f) shows the effect of having noisy response system data.

Application to higher-order system

Consider the system with poles at -0.5, -8 and -12, given by equation 7

$$G(s) = \frac{7s+20}{s^3+20.5s^2+106s+48} \quad (7)$$

If we applying the above approach to this system, it yields a first order system with a pole at -0.521.

$$G(s) = \frac{0.42}{1.92s+1} \quad (8)$$

Figure 7 shows the results of this approach applied to the third order system. Figure 7 (a) shows a comparison of this approximation to the step response of the third order actual system and as can be seen, for this input condition, they are quite close.

The Hough controller

We can use the parameters obtained by the simpler model to generate a controller for the system. For example, adding a proportional integral controller to the first order system

$$c(s) = k_p + \frac{k_i}{s} \quad (9)$$

and closing the loop, gives a closed-loop transfer function

$$G_{CL}(s) = \frac{k(k_p s+k_i)}{\tau s^2+(1+kk_p)s+kk_i} \quad (10)$$

We can specify the closed-loop second order dynamics by setting the denominator to $s^2 + 2\xi\omega_n s + \omega_n^2$ and choosing a value of ξ and ω_n.

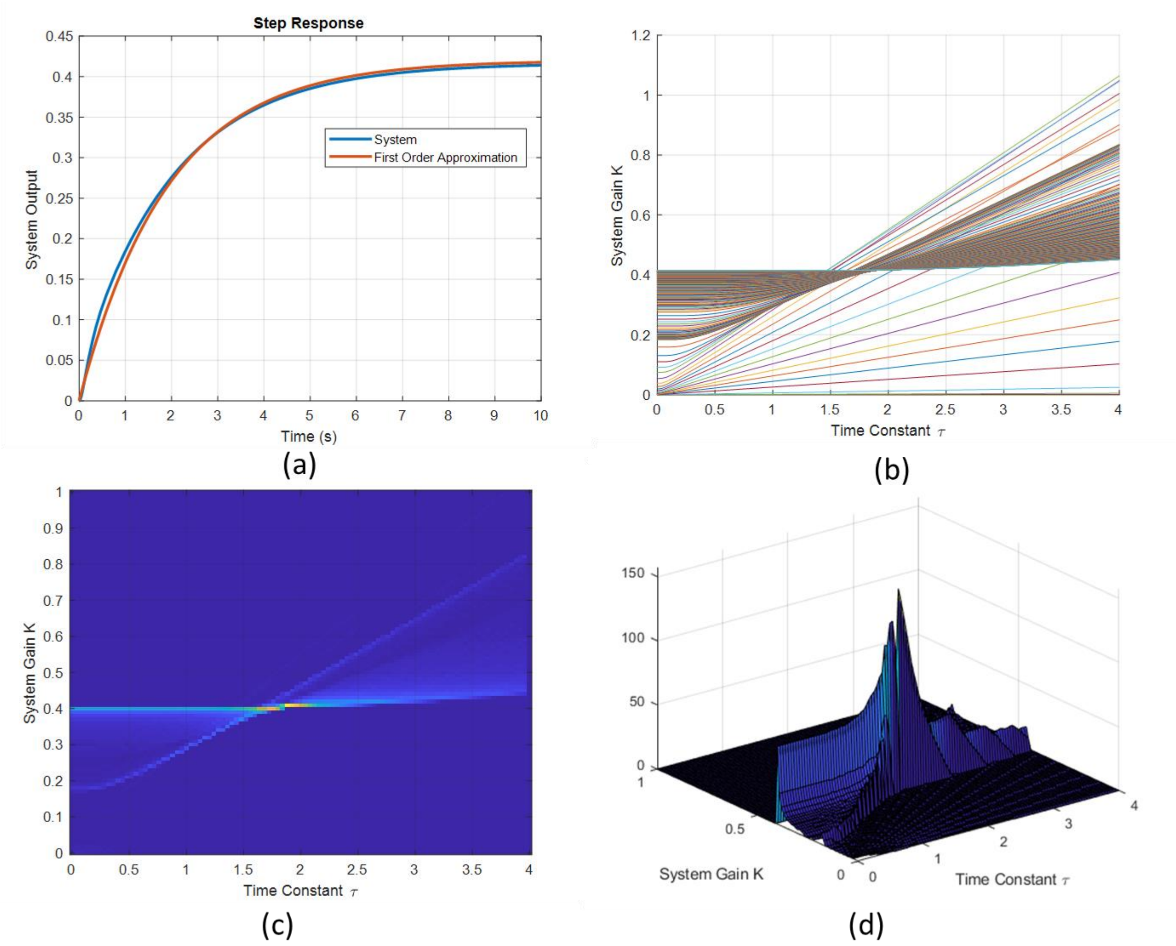


Figure 7. Parameter estimation of a higher-order system approximated by a first order system. (a) the step response of the system and the first order approximation obtained (b) system parameter space for the step response (c)-(d) the accumulator in parameter space.

For example, choosing $\xi = 0.7$ and $\omega_n = 1$ yields the response in figure 8, where

$$k_i = \frac{\tau\omega_n^2}{k} \quad \text{and} \quad k_p = \frac{2\xi\omega_n\tau - 1}{k} \tag{11}$$

Figure 8 show the actual closed-loop system response obtained compared to the lower-order system with which the controller was developed,

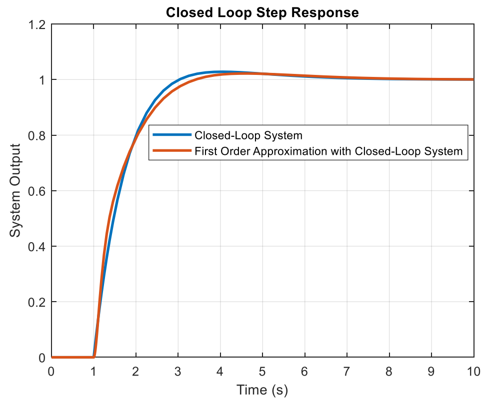


Figure 8. Closed-loop step response

An interesting alternative to determining the system parameters and then obtaining a controller is to determine the controller parameters directly. The Hough transform is applied to provide a direct mapping from the open-loop system step response to the controller parameters. We then have a map to controller parameter space (k_p, k_i) instead of mapping to the system coefficients (k, τ). Unfortunately, a closed form solution from step response to (k_p, k_i) parameter space could not be found.

The system parameters can similarly be obtained, using the Hough transforms, using the system step response of a first-order system with a zero. The system transfer function is given by equation 12.

$$G(s) = \frac{\tau_z s+1}{\tau_p S+1} \tag{12}$$

This can be rewritten as

$$G(s) = \frac{Bs+A}{S+A} \tag{13}$$

where $A = 1/\tau_p$ and $B = \tau_z/\tau_p$.

The time-domain system step response in this case is then given by

$$y(t) = 1 - (B-1)e^{(-At)} \tag{14}$$

Rearranging this gives

$$B = \left(\frac{y(t)-1}{e^{(-At)}}\right) + 1 \tag{15}$$

So for each value of (t,y) we can plot a line in (A,B) space. Figure 9 shows an example for a system with coefficients τ_z = 0.2 and τ_p =0.7. As can be seen, the parameters can be easily obtained, even with additive noise.

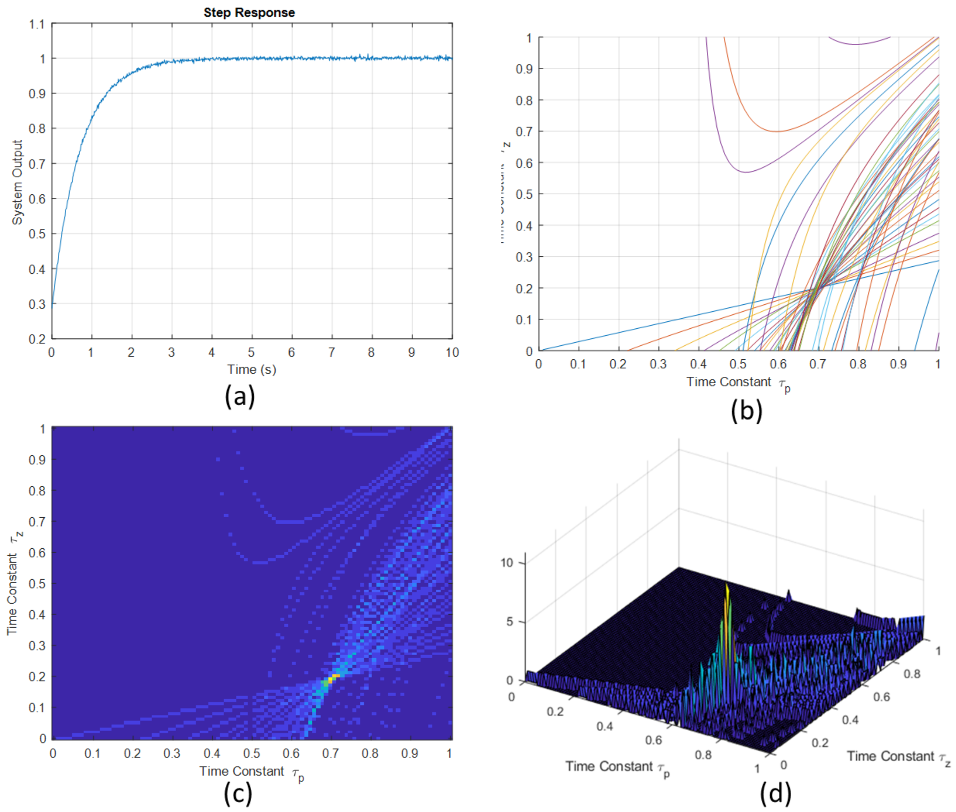


Figure 9. Step response for a first order pole zero system and Hough parameter estimation

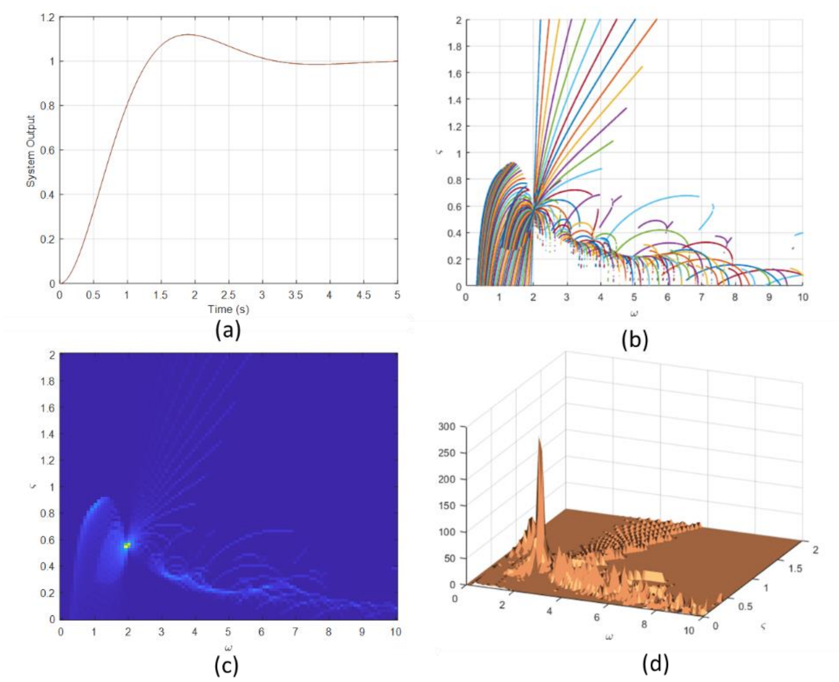


Figure 10. Open-loop step response for a second-order system and Hough parameter estimation

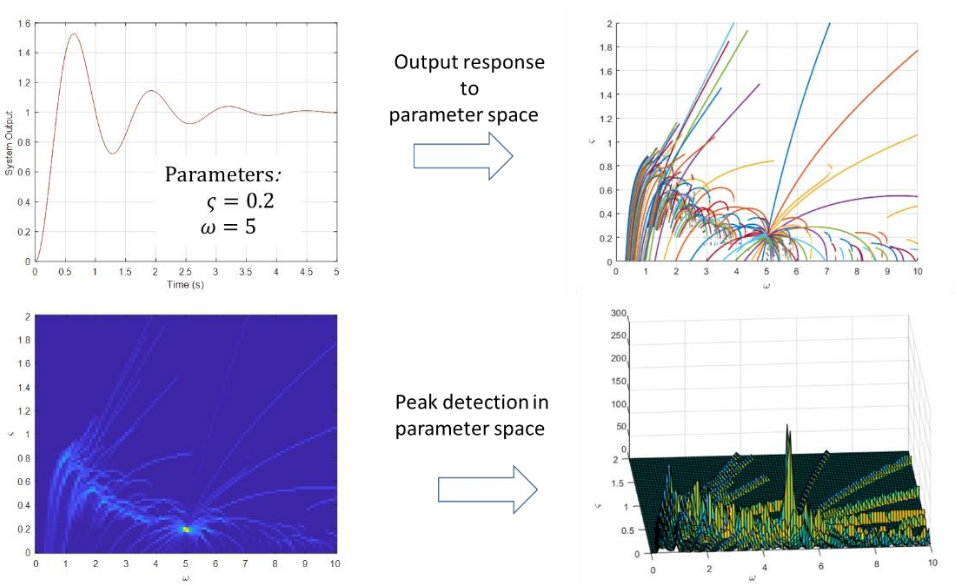


Figure 11. Open-loop step response for a second-order system and Hough parameter estimation

Figure 10 and 11 show the concept applied to determine the parameters of second-order systems for unit step response inputs .

Extension to arbitrary inputs

A wide range of systems have dominant first or second order dynamics and the above approach can be used on systems where a step response can be easily obtained conditions. In this section, we extend the approach to more general inputs, such as where the system may be in conditions of continuous closed-loop operation.

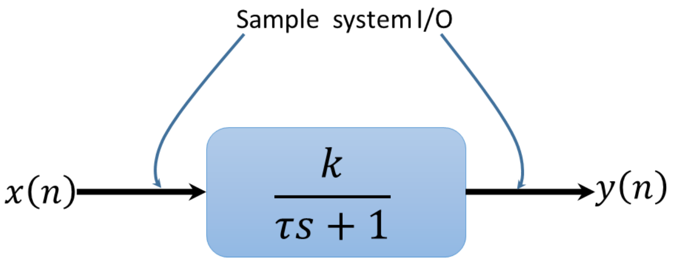


Figure 12. Sampling the system

Consider again the first order continuous system

$$G(s) = \frac{k}{\tau s+1} \tag{16}$$

The discrete-time system equivalent (since we are sampling from the Continuous-time System) is

$$G(z) = \frac{k'}{z-p} \tag{17}$$

With sampling rate T_s, the two parameters p and k' are given by

$$k' = k\left(1 - e^{\left(-\frac{T_s}{\tau}\right)}\right) \quad \text{and} \quad p = e^{\left(-T_s/\tau\right)} \tag{18}$$

As a difference equation, this is equivalent to

$$\mathrm{y}(n+1) = \mathrm{p}y(n) + k'x(n) \tag{19}$$

Rearranging the above equations, we get

$$k' = -\mathrm{p}\left(\frac{y(n)}{x(n)}\right) + \left(\frac{\mathrm{y}(n+1)}{x(n)}\right) \tag{20}$$

which can be seen as a straight line in (p, k') parameter space. Obviously, null input conditions need to be avoided to prevent divide by zero conditions. The Hesse normal form can again be used, in which case

$$r = \mathrm{x}\cos(\theta) + y\sin(\theta) \tag{21}$$

From θ and r

$$k' = \frac{r}{\sin(\theta)} \qquad p = -\cot(\theta)$$

$$\mathrm{k} = \frac{K'}{(1-p)} \qquad \tau = -\frac{T_s}{\log(-\cot(\theta))} \tag{22}$$

We are obtaining the (discrete-time) system parameters from sampling the systems input and output, as shown in figure 12, in this case for general input conditions. Figure 13 shows this approach applied to the first-order system we used earlier with $k = 2,\ \tau = 0.7$. As can be seen the Hesse approach helps to obtain the parameters.

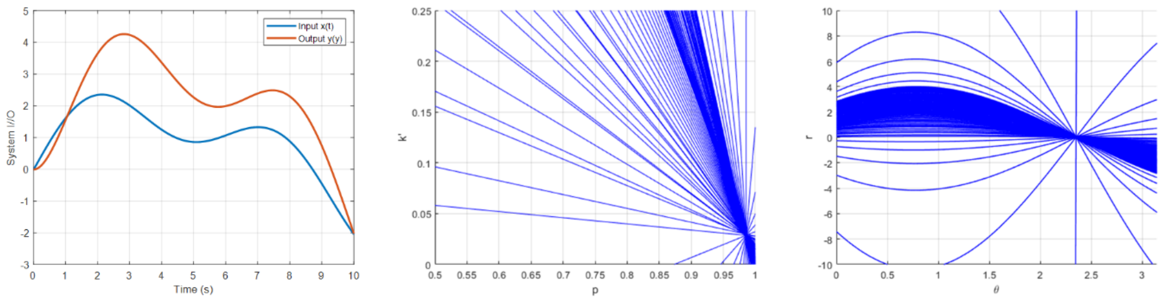


Figure 13. First-order system and Hough parameter estimation with general input conditions

Consider the continuous system, with an additional integrator given by

$$G(s) = \frac{k}{s(\tau s+1)} \tag{23}$$

This system is open-loop unstable, so need to sample it in a closed-loop system, at the input and output of the plant

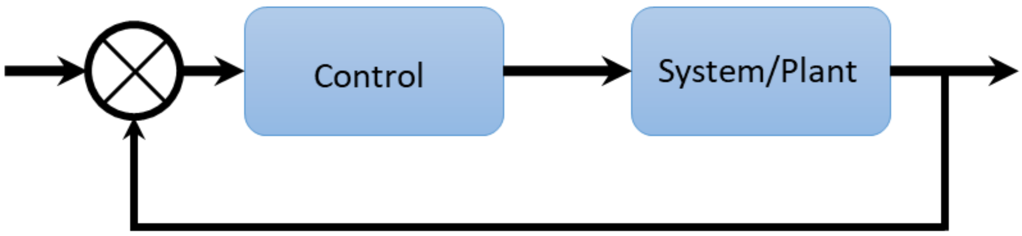


Figure 14

A discrete-time equivalent model for this system is

$$\mathrm{G(z)} = \frac{\mathrm{k'z^{-1}}}{\mathrm{a_1+a_2z^{-1}+a_3z^{-2}}} \tag{24}$$

Where $a_1 = 1$, $a_2 = -(1 + a_3)$, $a_3 = e^{\left(-T_s/_\tau\right)}$, $k' = (1 - a_3)T_s k$

The two parameters a_3 and k', again give the equation of a straight line

$$k' = a_3\left(\frac{y(n-2)-y(n-1)}{x(n-1)}\right) + \left(\frac{y(n)-y(n-1)}{x(n-1)}\right) \tag{25}$$

and so can be solved in a similar manner

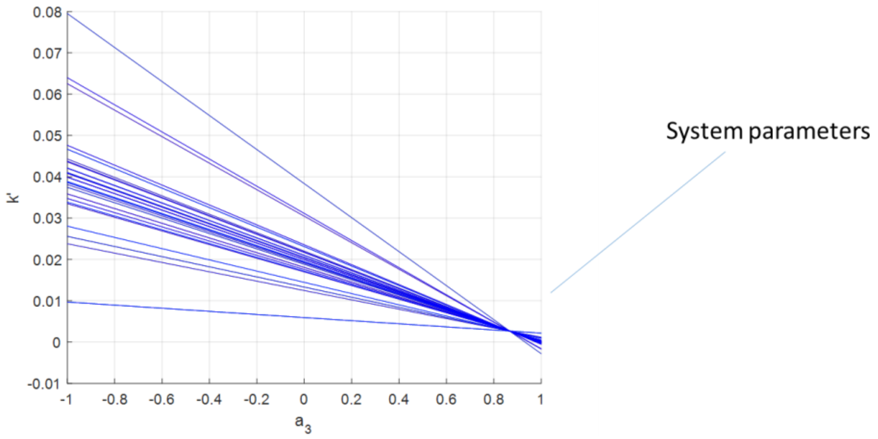


Figure 15

If the system is a second order system

$$G(s) = \frac{\omega_n^2}{s^2+2\varsigma\omega_n s+\omega_n^2} \tag{26}$$

with poles

$$p_{a1} = -\varsigma\omega_n + \omega_n\sqrt{\varsigma^2 - 1} \qquad p_{a2} = -\varsigma\omega_n - \omega_n\sqrt{\varsigma^2 - 1}$$

The inputs and outputs can be sampled, so a discrete-time equivalent needs to be considered

$$H(z) = K\frac{b_1+b_2z^{-1}+b_3z^{-2}}{a_1+a_2z^{-1}+a_3z^{-2}} \tag{27}$$

With poles $p_1 = e^{p_{a1}T_s}$ and $p_2 = e^{p_{a2}T_s}$,where

$b_1 = 1$ $\quad b_2 = 2$ $\quad b_3 = 1$

$a_1 = 1$ $\quad a_2 = -(p_1 + p_2)$ $\quad a_3 = p_1p_2$

$K = \frac{1+a_2+a_3}{4}$

As a difference equation, this can be written,

$$y(n) = -a_2y(n-1) - a_3y(n-2) + Kx(n)+2Kx(n-1)+ Kx(n-2)$$

and reformed as

$$y(n) = -a_2y(n-1) - a_3y(n-2) + K\big(x(n)+2x(n-1)+x(n-2)\big)$$

$$y(n) = -a_2y(n-1) - a_3y(n-2) + \frac{1+a_2+a_3}{4}\big(x(n)+2x(n-1)+x(n-2)\big)$$

$$4y(n) = -a_24y(n-1) - a_34y(n-2) + (1 + a_2 + a_3)\big(x(n)+2x(n-1)+x(n-2)\big)$$

Let $X = \big(x(n)+2x(n-1)+x(n-2)\big)$, then

$$(4y(n) - X) = a_2(X - 4y(n-1)) + a_3\big(X - 4y(n-2)\big)$$

Which is a straight line in (a_2, a_3) space. Although because there is a relationship between a_2 and a_3 this solution doesn't fill the space.

Figure 17 was obtained by sweeping w and solving an optimization to obtain sigma

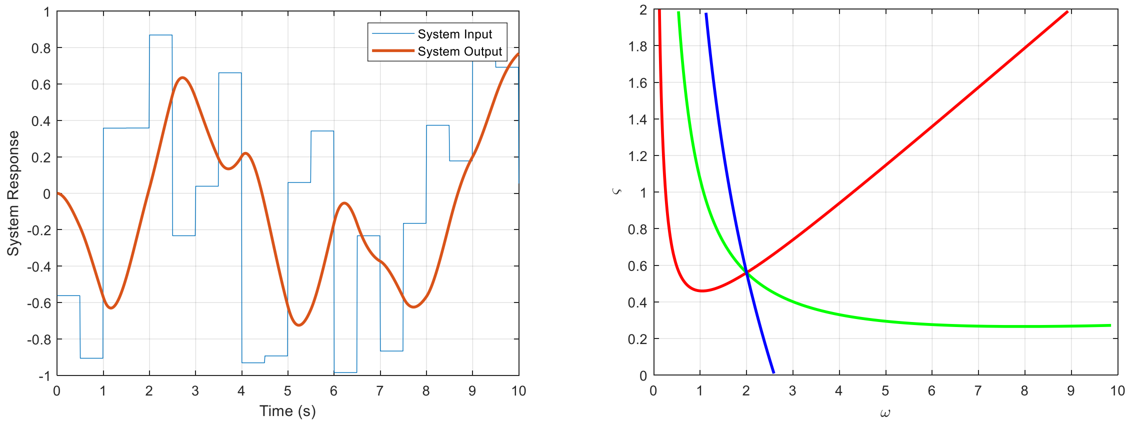


Figure 17

Once we have obtained the system parameters, we can use this to adapt the system controller, as shown in figure 18, or as before try to obtain the controller parameters directly.

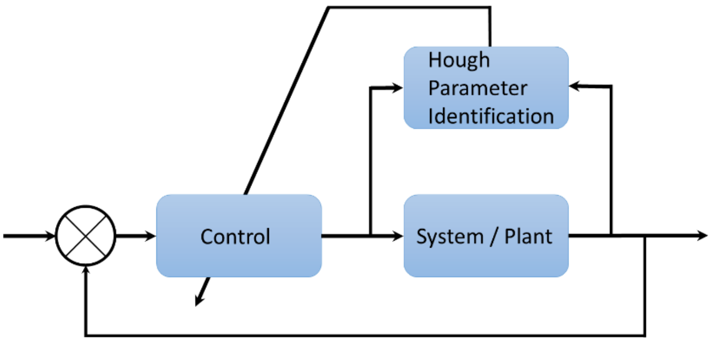


Figure 18. An adaptive Hough Controller

Conclusions

This paper investigates two approaches to controller development that exploit the properties of the Hough transform. The first approach is to use the concept of the Hough transform to estimate the parameters of the system or process to be controlled. The second approach uses the Hough transform to estimate suitable controller parameters directly for the system. The Hough transform is very efficient for estimating a low number of parameters (three parameters or less). Many industrial dynamic processes have dominant first or second order dynamics and many commonly used controllers, such as a PID controller have relatively low number of parameters. This makes the Hough controller concept a feasible alternative to other methodologies. Challenges remain around the bin size used. If the regions are too small then they will be indistinguishable from the neighbouring bins until enough data is available. This adaptation could affect the usage for adaptive controllers in noisy environments. One possible alternative to examine is the use of the Radon transform in place of the Hough transform.

Pattern recognition is a large research area and this paper has only considered one, relatively simple technique, from this domain for applications in systems identification and control. It is envisioned that many applications of pattern recognition await development within the field of control systems engineering. Suitable areas for investigation are system identification, fault detection and isolation and as part of a systems state of health assessment.

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Change log

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22.2.2023 corrected second order system poles formula to remove the s and the 2

Added 17.5.2021

added a reference to a basic control engineering test book

added a reference to book on Patten recognition control

added reference for paper on Patten recognition control

Added 26.5.2021

fixing text.

Figures 1 – 3 generated by HuffLines.m

Figures 5 – 6 generated by Hough.m and huff_fos.slx

Figure 7 generated with hom.m and huff_hom.slx

Figure 7-8 generated with hom.m and huff_hom.slx

Figure 9 generated with hough1.m and huff_fosz.slx

Figure 11 Hough2 ?

Figure 13 Hough_DiscreteTimeExample

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