Towards a Hough Controllers

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Outline

- Hough transform
  - Algorithm and Line Detection

Application of the Hough Transform to control systems

- System Identification
  - Proof of concept
    - Step response system parameter identification using the Hough Transform
  - System identification for generalized inputs
- System Control
  - Adaptive pole placement

Hough transform

- Invented by Paul Hough
- Patented in 1962

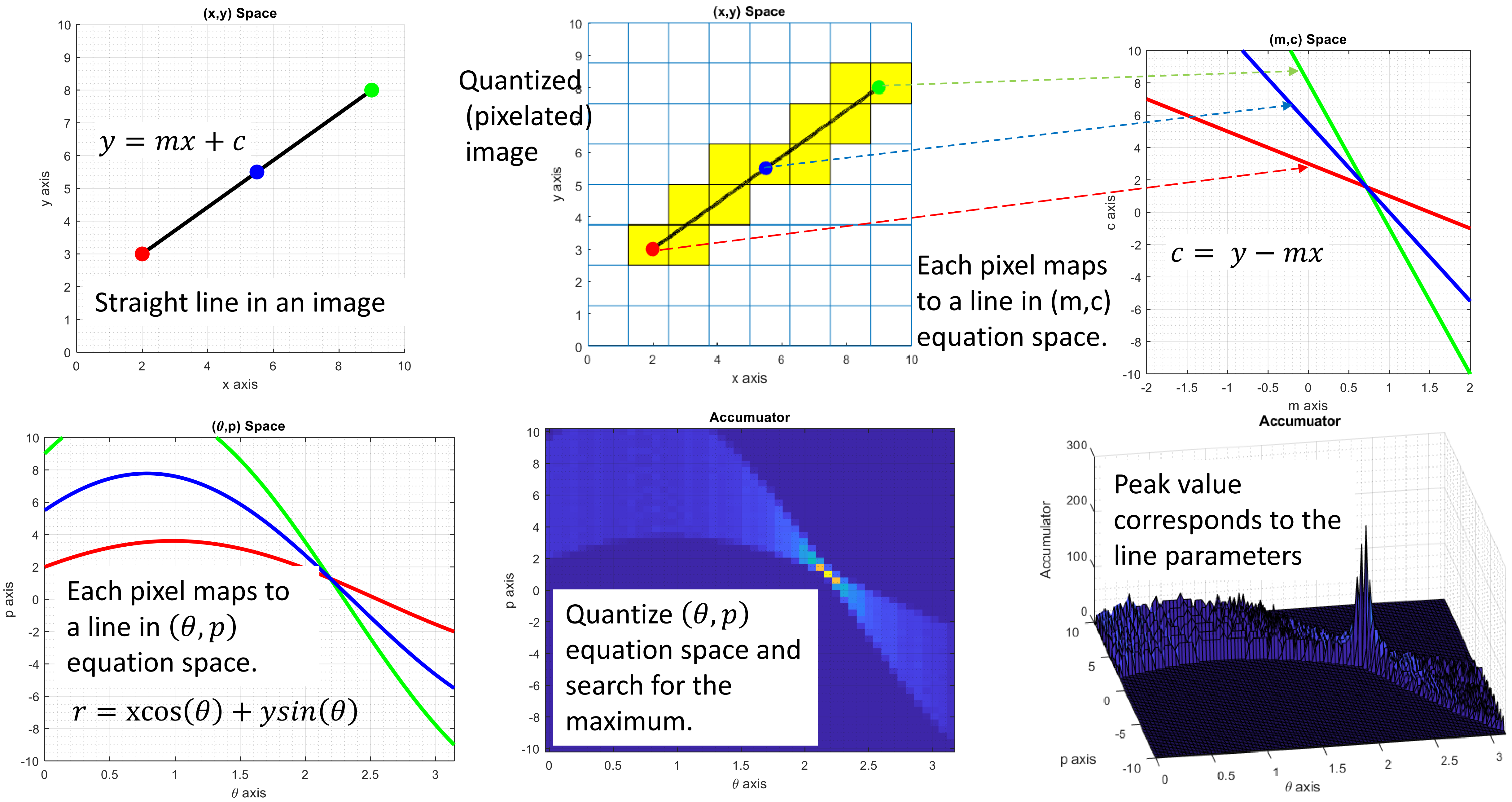
    U.S. Patent 3,069,654 “Method and Means for Recognizing Complex Patterns".
- Translate from image space to parameter space
- Simple technique
- Can handle missing data
- Works well for low dimensional objects (lines and circles)
- Disadvantage: Large storage

Algorithm for line detection

- For each data point in image get a line in parameter space
- If the line goes into a quantized area in parameter space value incremented in an accumulator
- This is repeated for each point in the image
- Get an image in Hough space
- Maximum value of the accumulator is the coordinates of the line

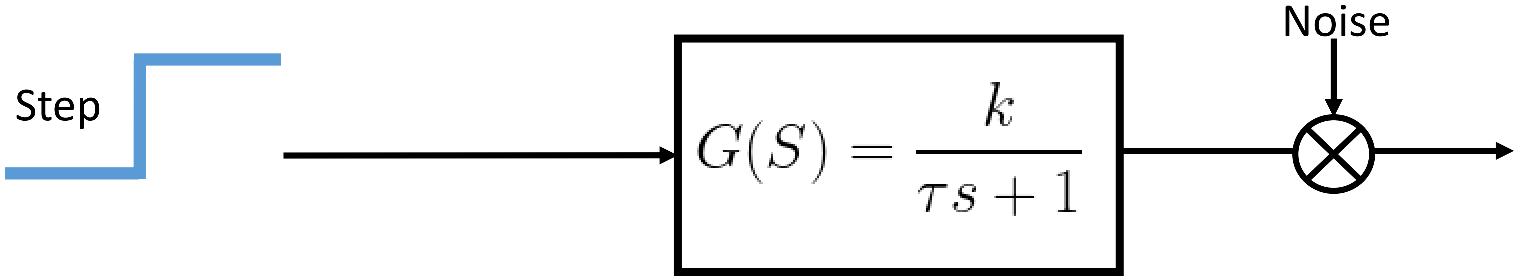
Maps from image space to parameter space

Hough transform for a straight line

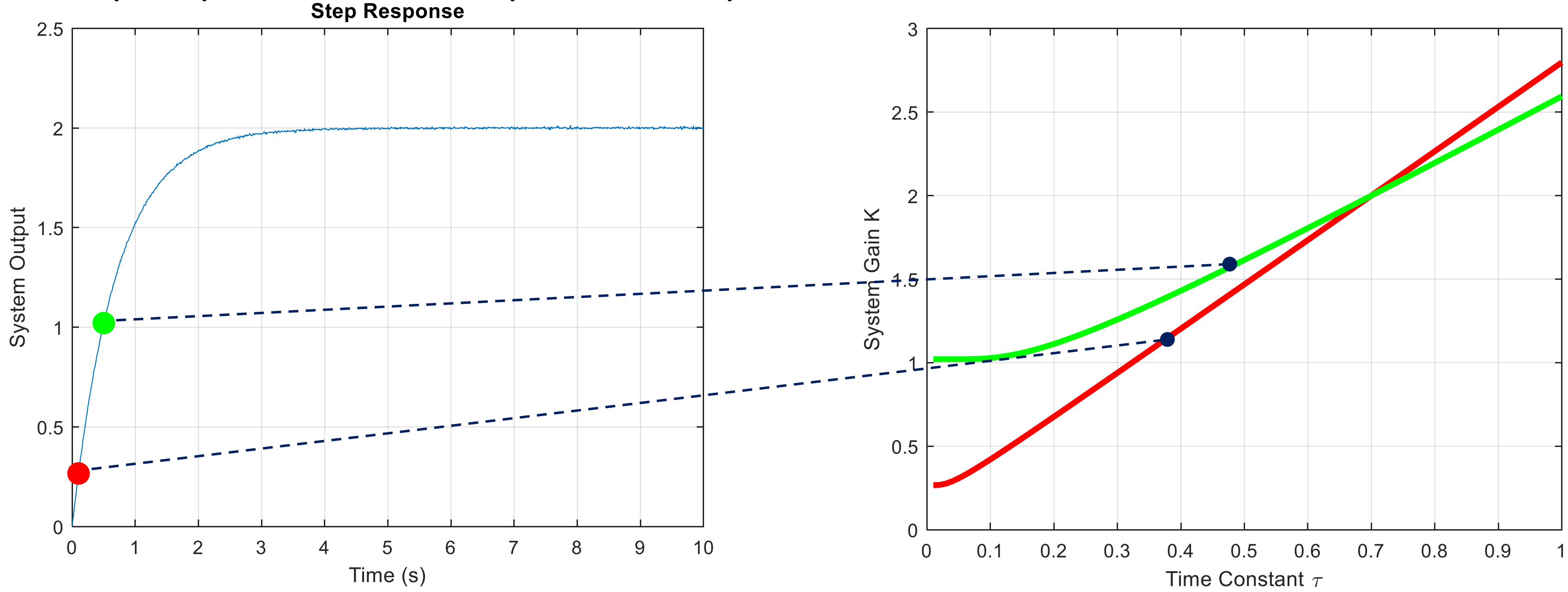


Can we apply the concept for control?

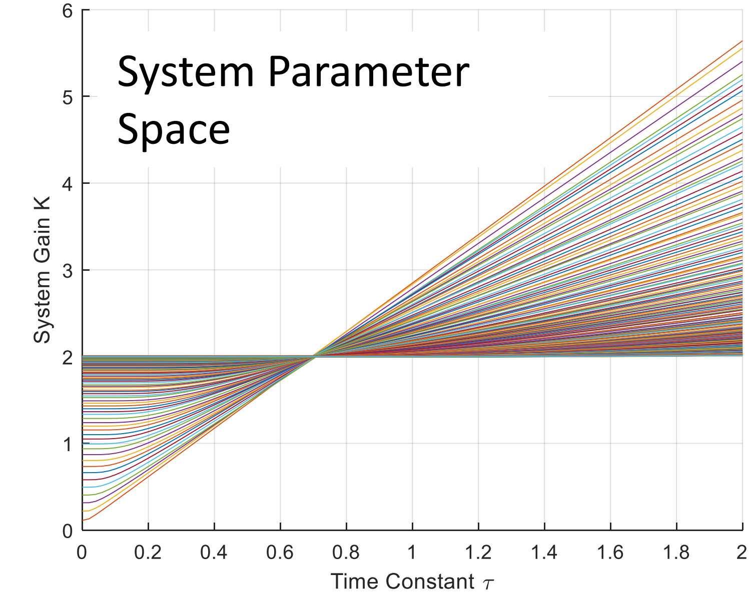
Simple example:
Parameter estimation for a first order system

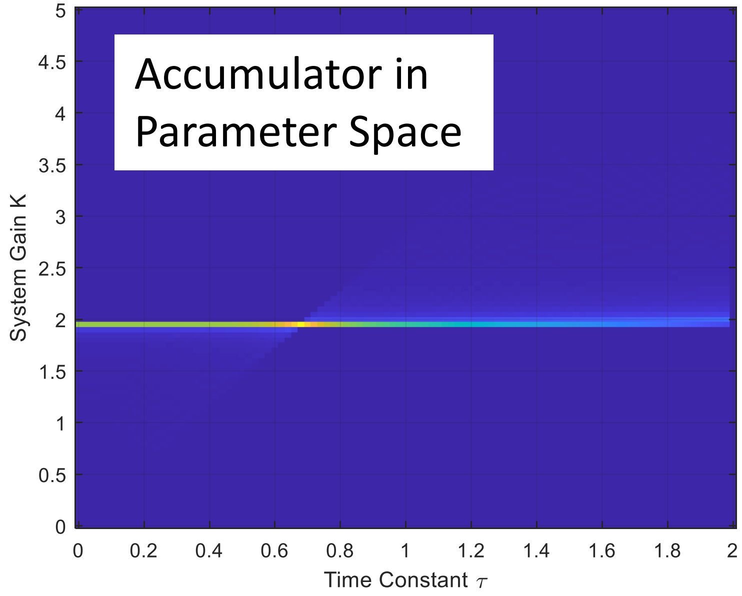


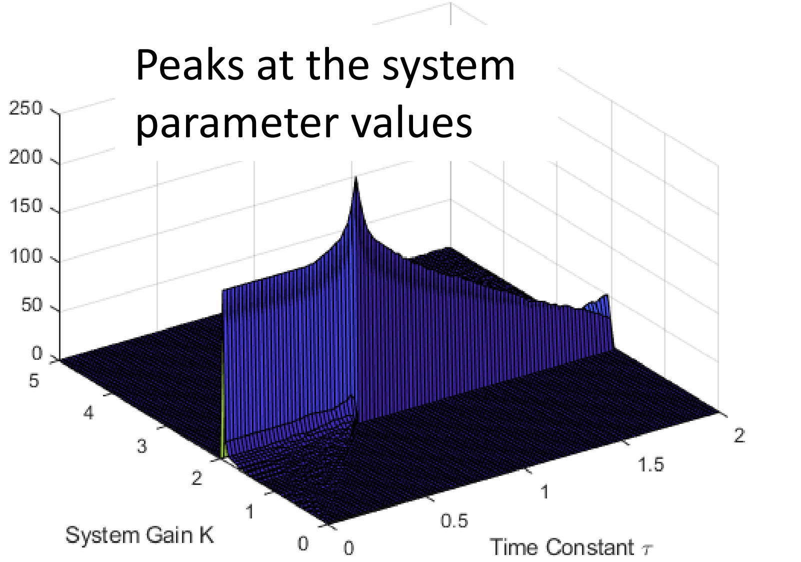
Each data point on the step response is a line in parameter space



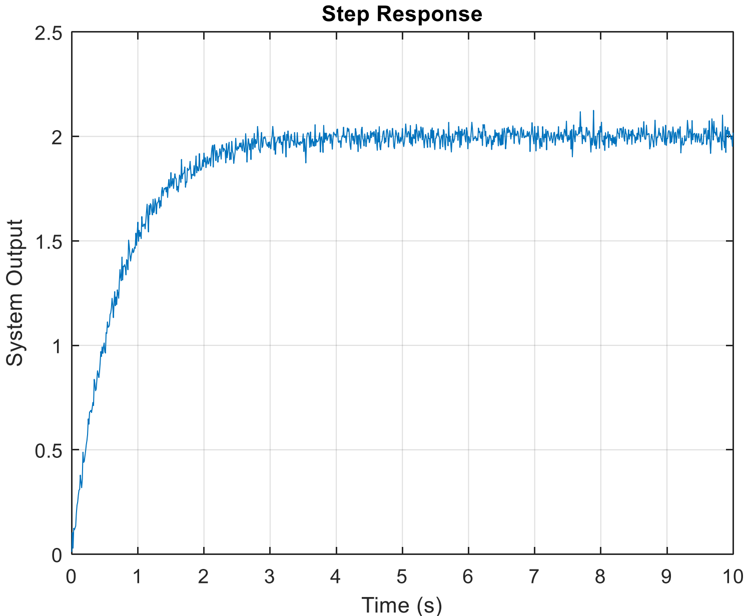
Maps from system response to system parameters space

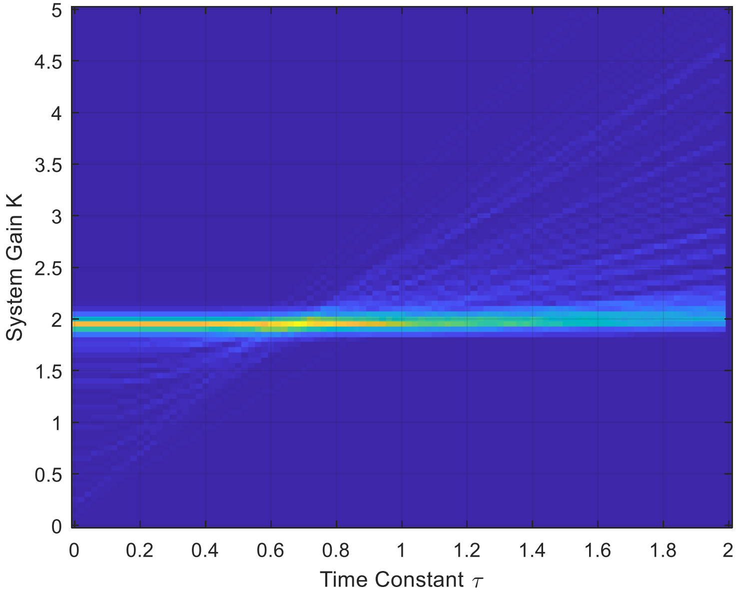


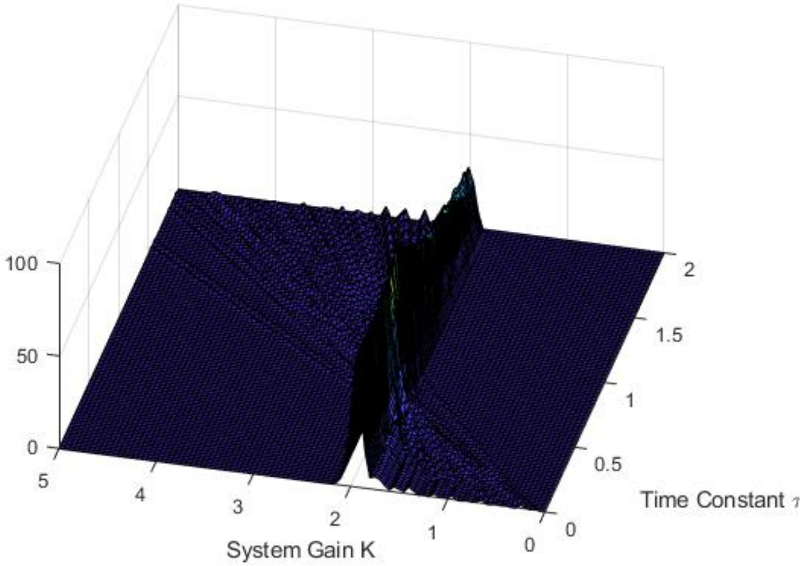




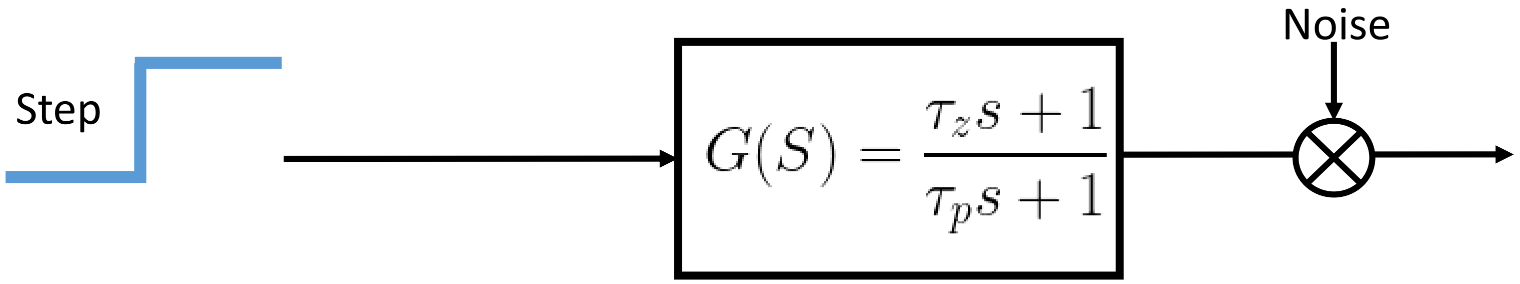
Robustness to noise

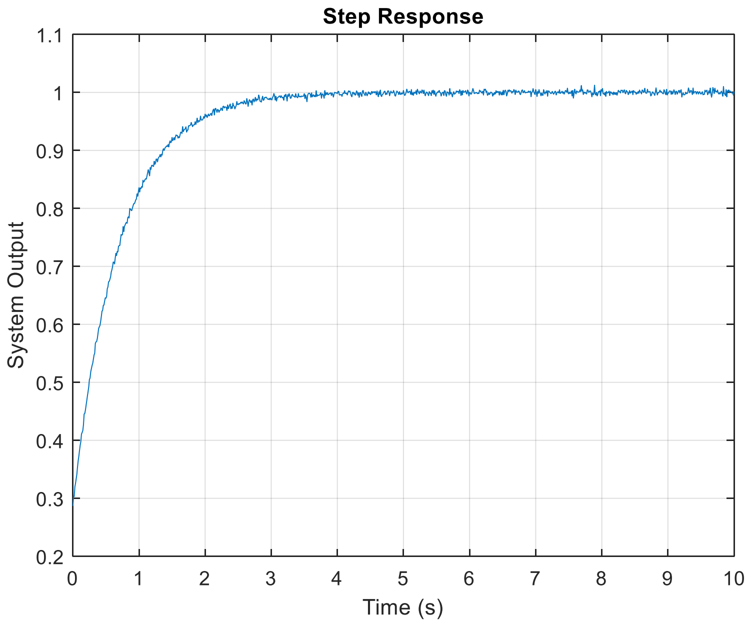




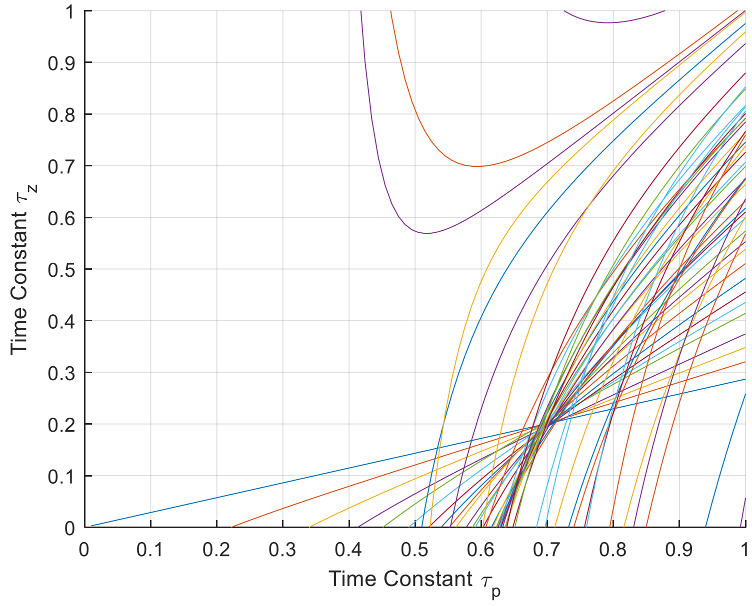


Applied to a control System

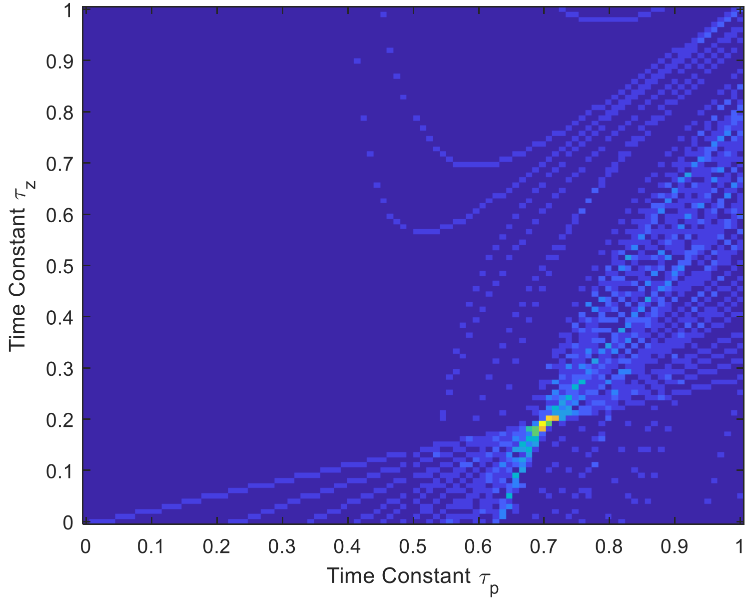


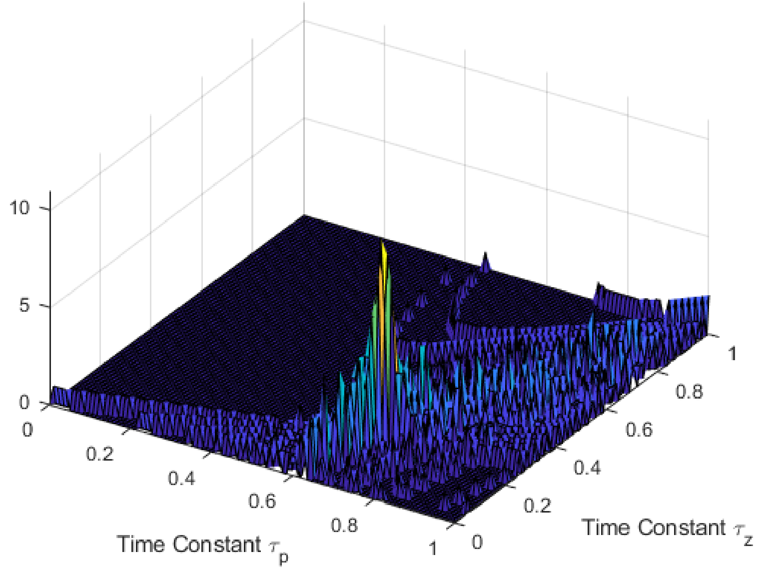


Parameter space

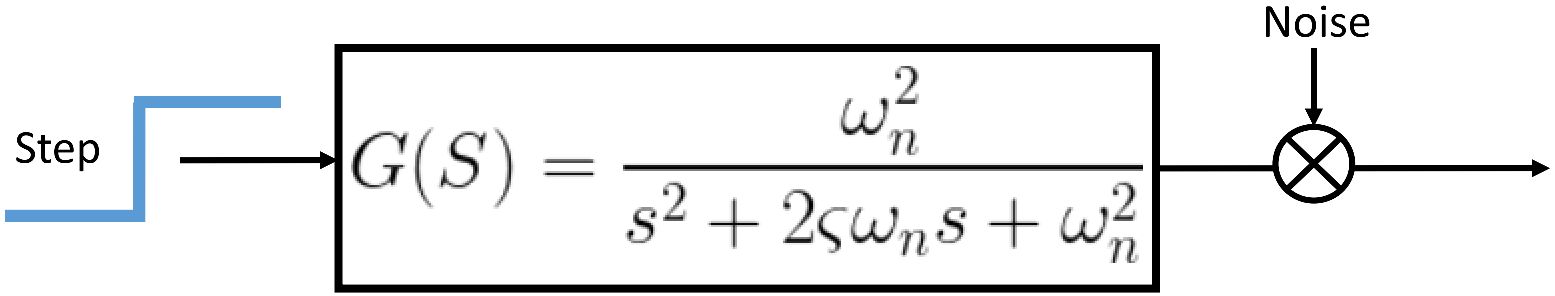


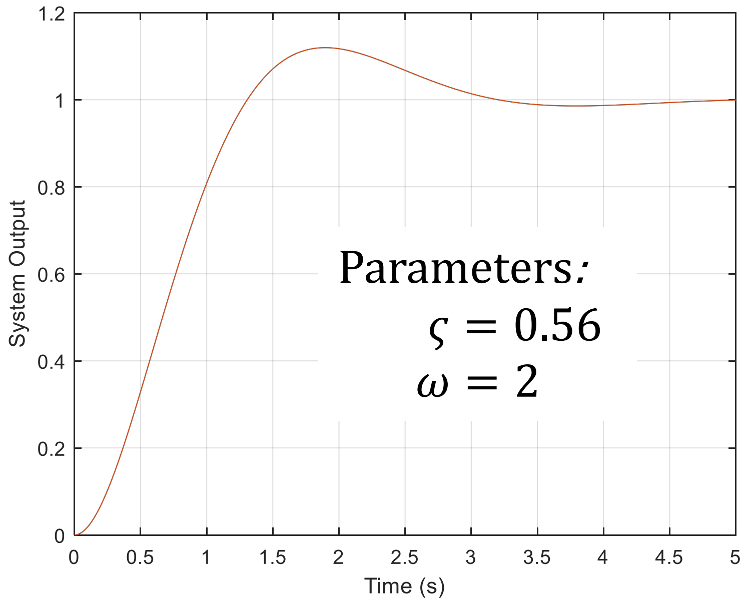
Accumulator in parameter space

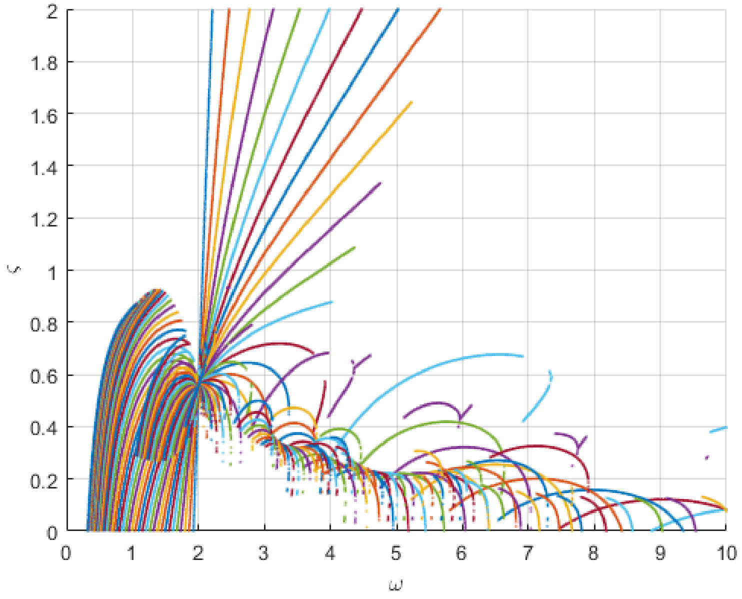


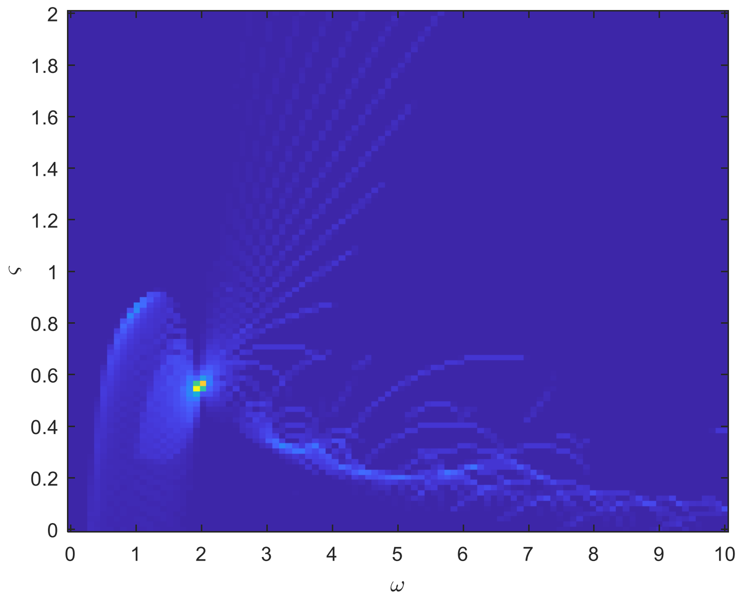


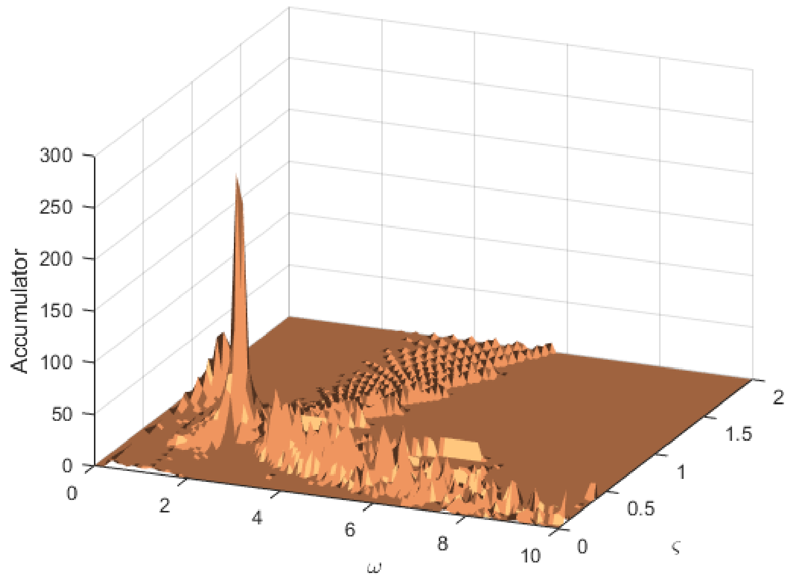
Second-order system



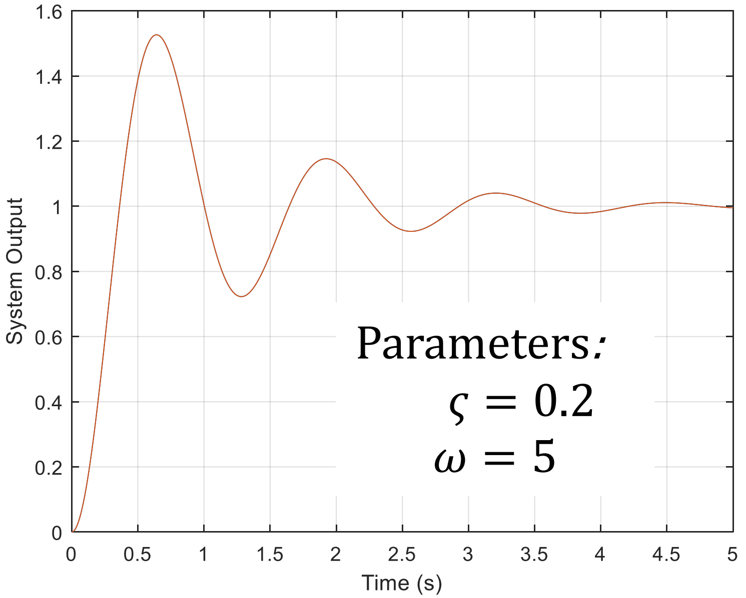




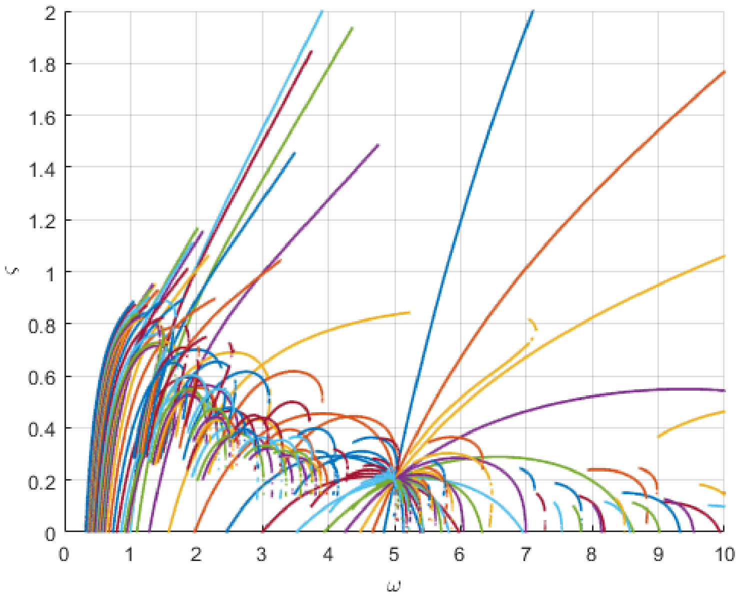


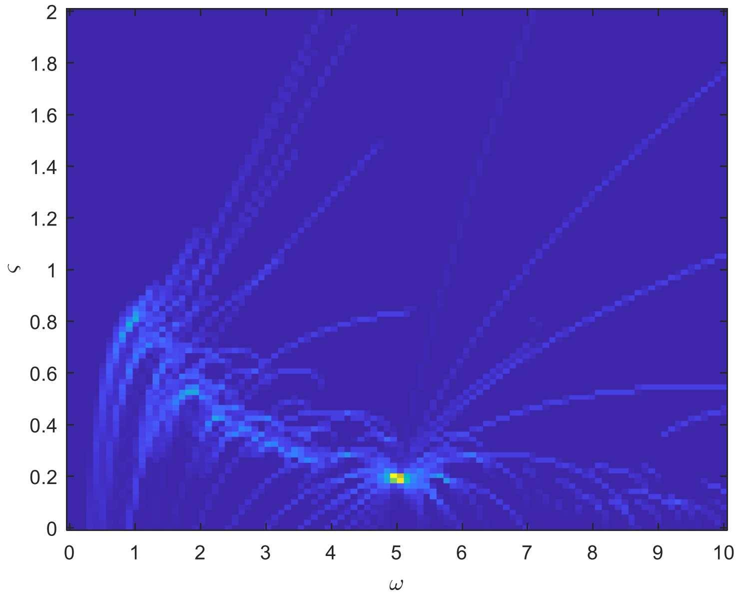


Second-order system

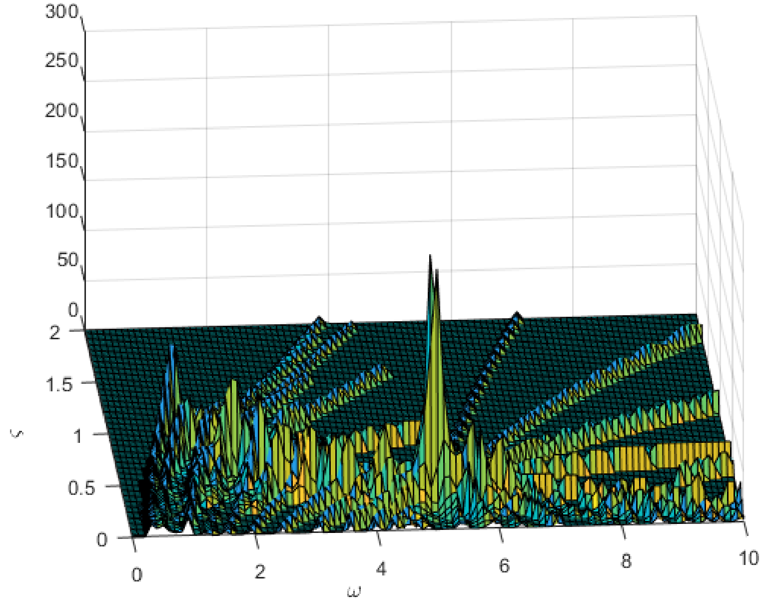


Output response
to
parameter space





Peak detection in
parameter space



Can we generalize this to determine parameters for arbitrary inputs ?

First order system

Continuous-time System

$$G(s) = \frac{k}{\tau s + 1}$$

Discrete-time system equivalent (since we are sampling from the Continuous-time System)

$$G(z) = \frac{k'}{z - p} \qquad k' = \left(1 - e^{\left(-\frac{T_s}{\tau}\right)}\right) \qquad p = e^{\left(-T_s/\tau\right)}$$

Two parameters p and k'

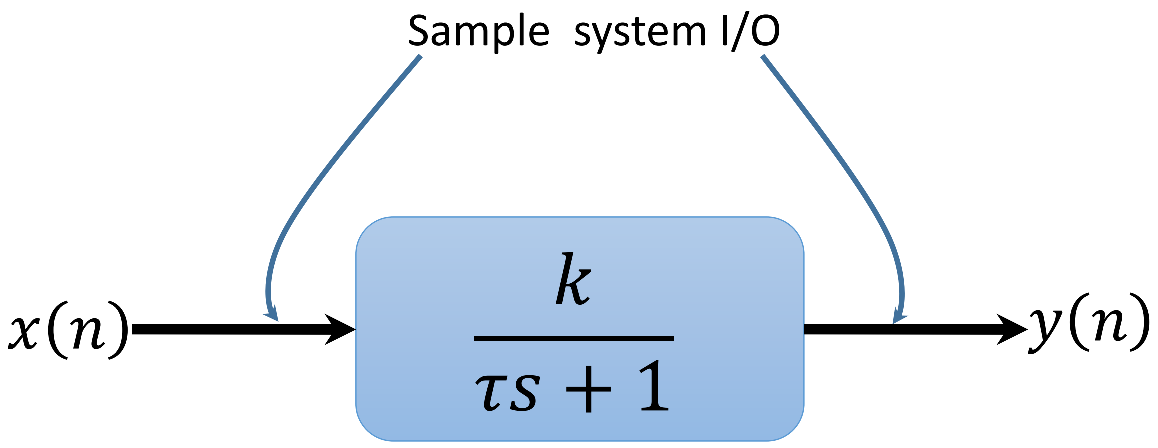
$$y(n+1) = py(n) + k'x(n)$$

Equation is a straight line in p and k'

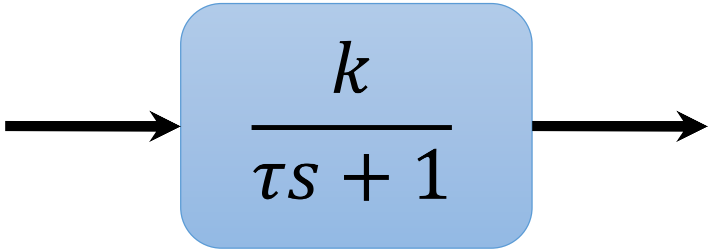
$$k' = -p\left(\frac{y(n)}{x(n)}\right) + \left(\frac{y(n+1)}{x(n)}\right)$$

Use the Hesse normal form

$$r = x\cos(\theta) + y\sin(\theta)$$



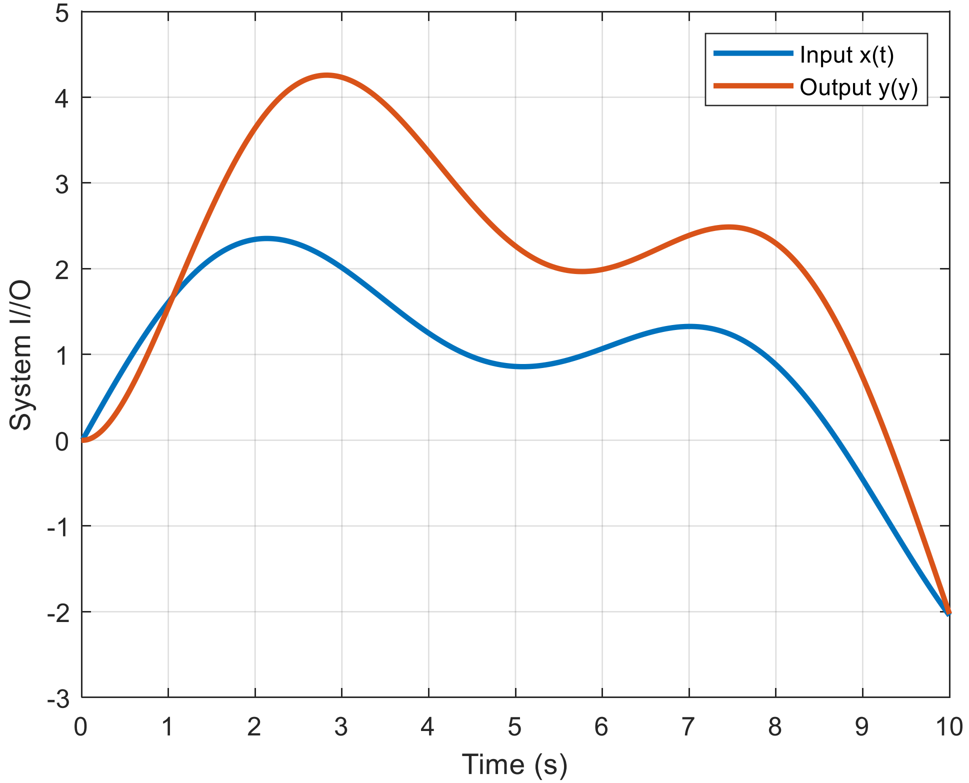
First order system continued

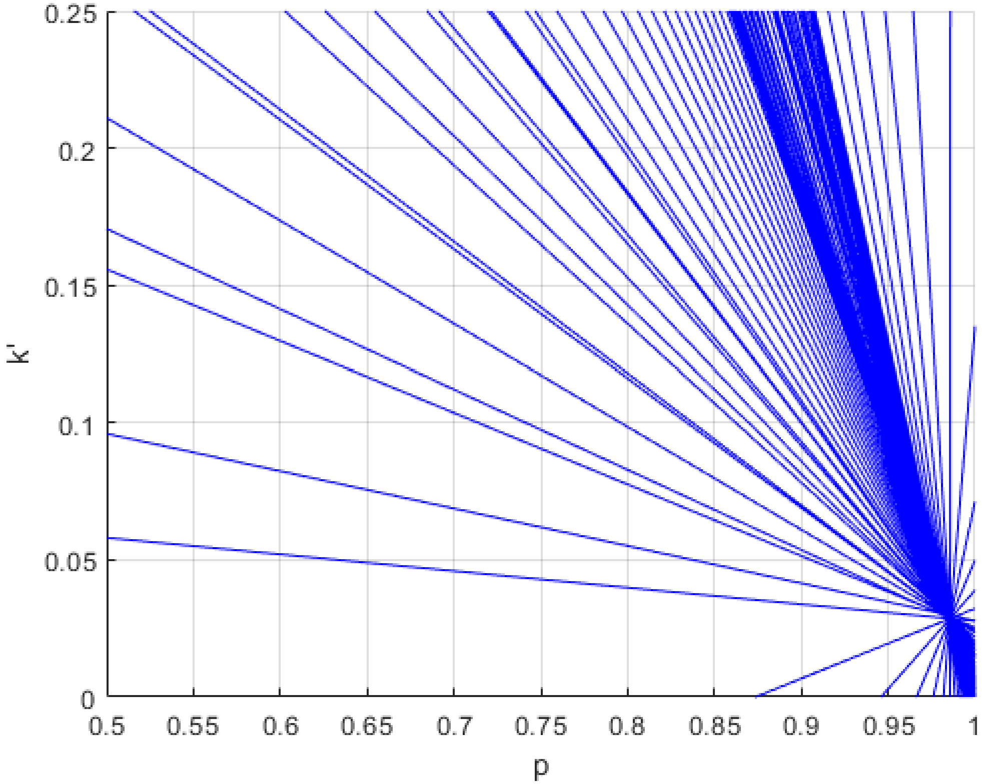


From HT we obtain r, θ

$$k' = \frac{r}{sin(\theta)} \qquad p = -cot(\theta)$$

Using the same parameters as we did for the step function (Tp = 0.7; Kp=2)

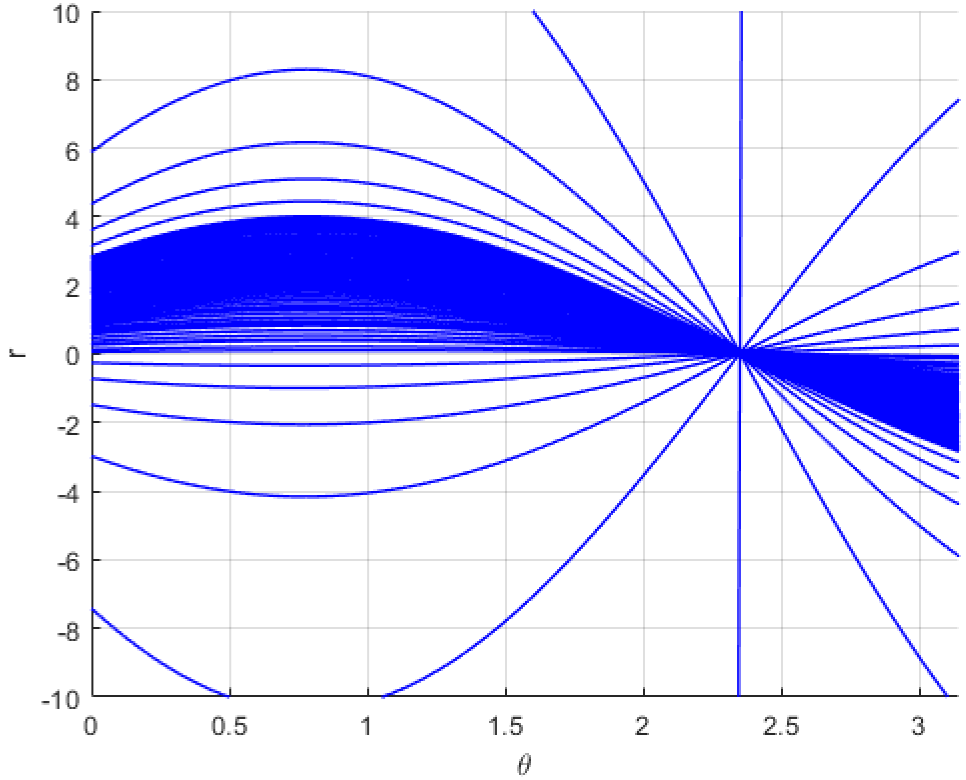




$$k' = 0.0284$$

$$p = 0.9858$$

First order system continued



$$\theta = 2.3491$$

$$r = 0.0202$$

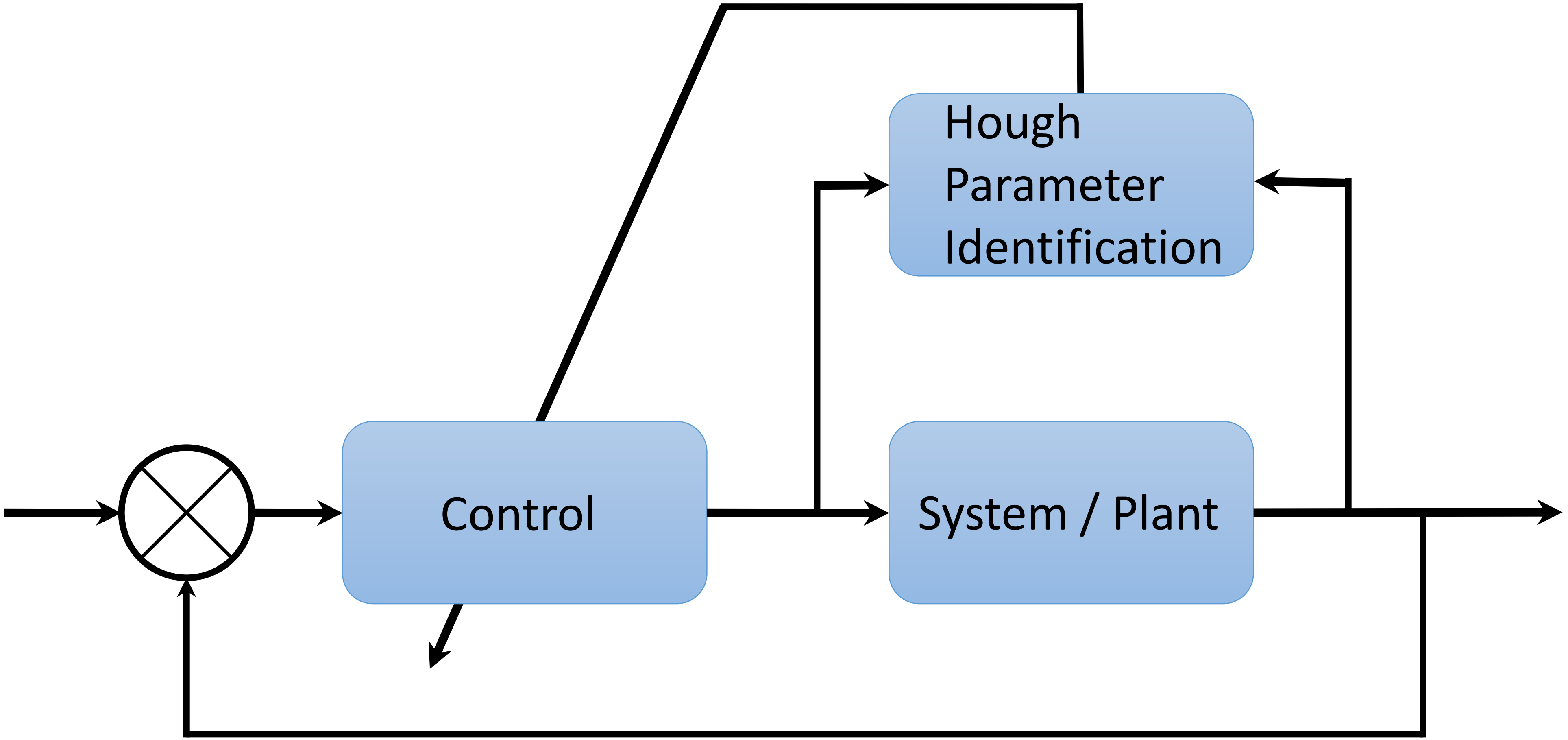
$$p = -cot(\theta)$$

$$k' = \frac{r}{sin(\theta)}$$

$$\tau = -\frac{T_s}{log\left(-cot(\theta)\right)}$$

$$\mathrm{k} = \frac{K'}{(1-p)}$$

Hough Controller



Second order system

Continuous time systems

$$G(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

With poles

$$p_{a1} = -\varsigma\omega_n + \omega_n\sqrt{\varsigma^2 - 1}$$
$$p_{a2} = -\varsigma\omega_n - \omega_n\sqrt{\varsigma^2 - 1}$$

Discrete time equivalent

$$H(z) = K\frac{b_1 + b_2 z^{-1} + b_3 z^{-2}}{a_1 + a_2 z^{-1} + a_3 z^{-2}}$$

With poles

$$p_1 = e^{p_{a1}T_s}$$
$$p_2 = e^{p_{a2}T_s}$$

$b_1 = 1$ $\quad$ $b_2 = 2$ $\quad$ $b_3 = 1$

$a_1 = 1$ $\quad$ $a_2 = -(p_1 + p_2)$ $\quad$ $a_3 = p_1 p_2$

$$K = \frac{1 + a_2 + a_3}{4}$$

Second order system continued

Discrete time equivalent

$$y(n) = -a_2 y(n-1) - a_3 y(n-2) + Kx(n) + 2Kx(n-1) + Kx(n-2)$$

$$y(n) = -a_2 y(n-1) - a_3 y(n-2) + K\big(x(n) + 2x(n-1) + x(n-2)\big)$$

$$y(n) = -a_2 y(n-1) - a_3 y(n-2) + \frac{1 + a_2 + a_3}{4}\big(x(n) + 2x(n-1) + x(n-2)\big)$$

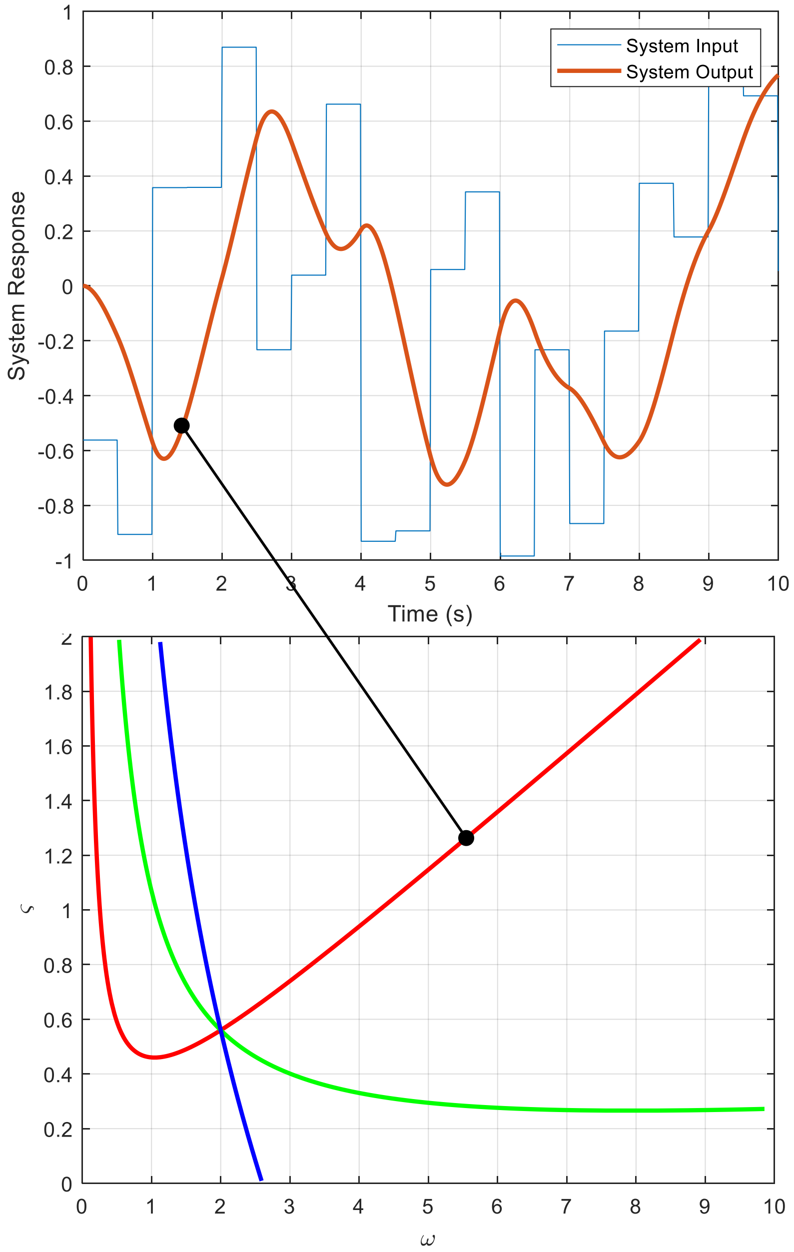
$$4y(n) = -a_2 4y(n-1) - a_3 4y(n-2) + (1 + a_2 + a_3)\big(x(n) + 2x(n-1) + x(n-2)\big)$$

Let $\mathrm{X} = \big(x(n) + 2x(n-1) + x(n-2)\big)$

$$(4y(n) - X) = a_2(X - 4y(n-1)\ ) + a_3\big(X - 4y(n-2)\big)$$

Which is a straight line in $(a_2\, , a_3)$ space

Second order system continued

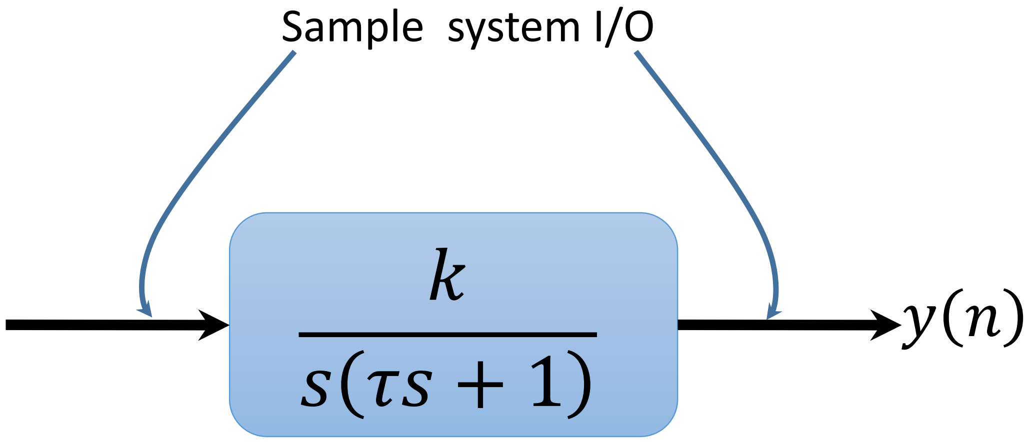


Although a straight line is a solution doesn't fill the space

Sweep w and solve an optimization to obtain sigma

Continuous-time System

$$G(s) = \frac{k}{s(\tau s + 1)}$$



Open loop unstable so need to sample in a closed-loop system

Discrete-time equivalent model

$$G(z) = \frac{k' z^{-1}}{a_1 + a_2 z^{-1} + a_3 z^{-2}}$$

$$a_1 = 1$$

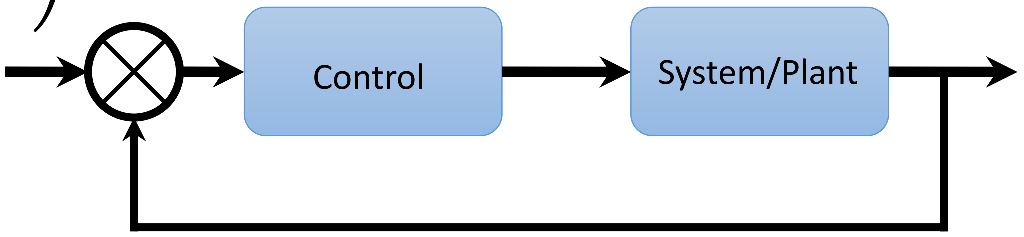
$$a_2 = -(1 + a_3\ )$$

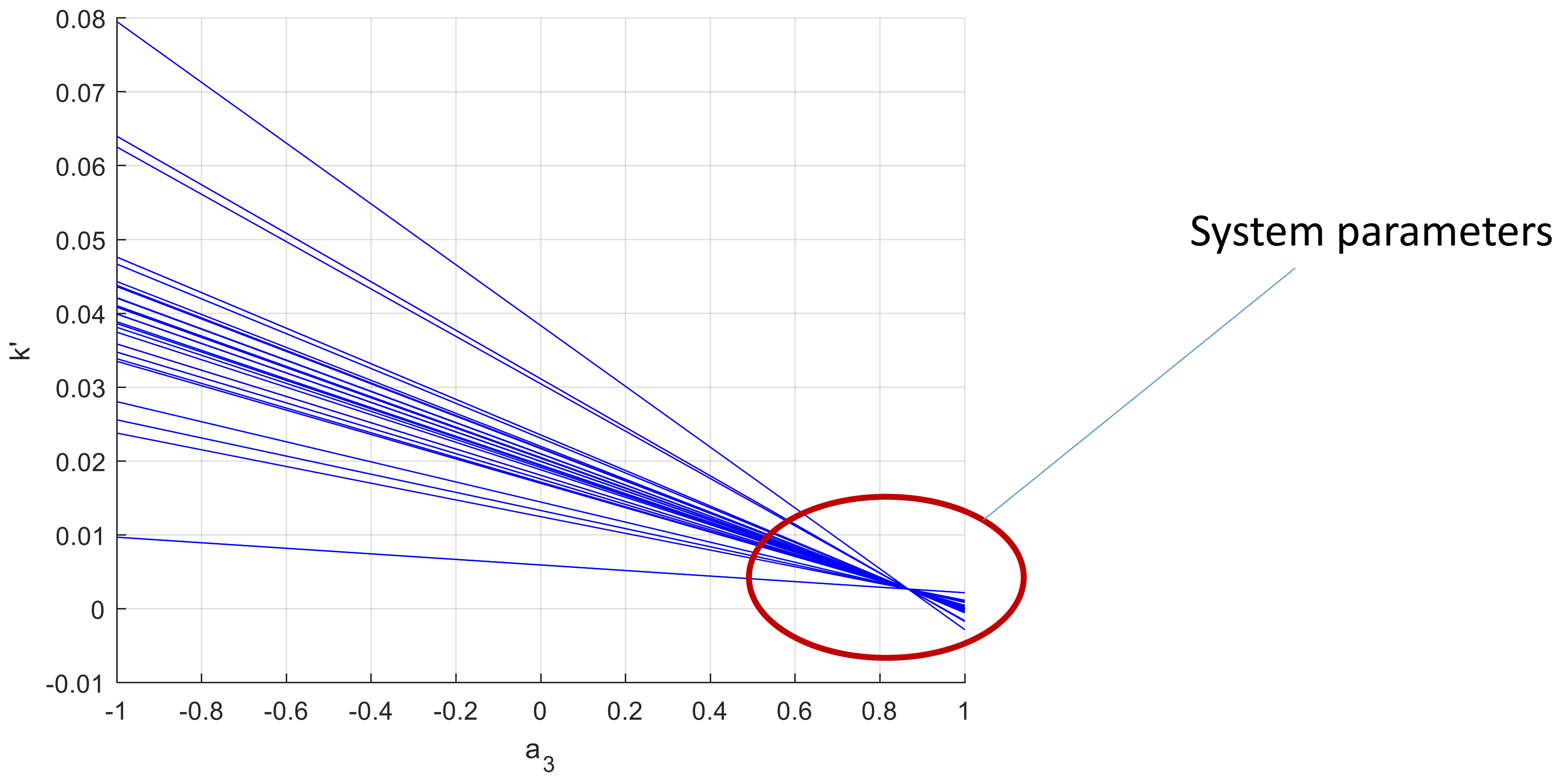
$$a_3 = e^{\left(-T_s/_\tau\right)}$$

$$k' = \ (1 - a_3) T_s k$$

Two parameters a_3 and k', give the equation is a straight line

$$k' = a_3 \left(\frac{y(n-2) - y(n-1)}{x(n-1)} \right) + \left(\frac{y(n) - y(n-1)}{x(n-1)} \right)$$





0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
k'
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
a_3
System parameters

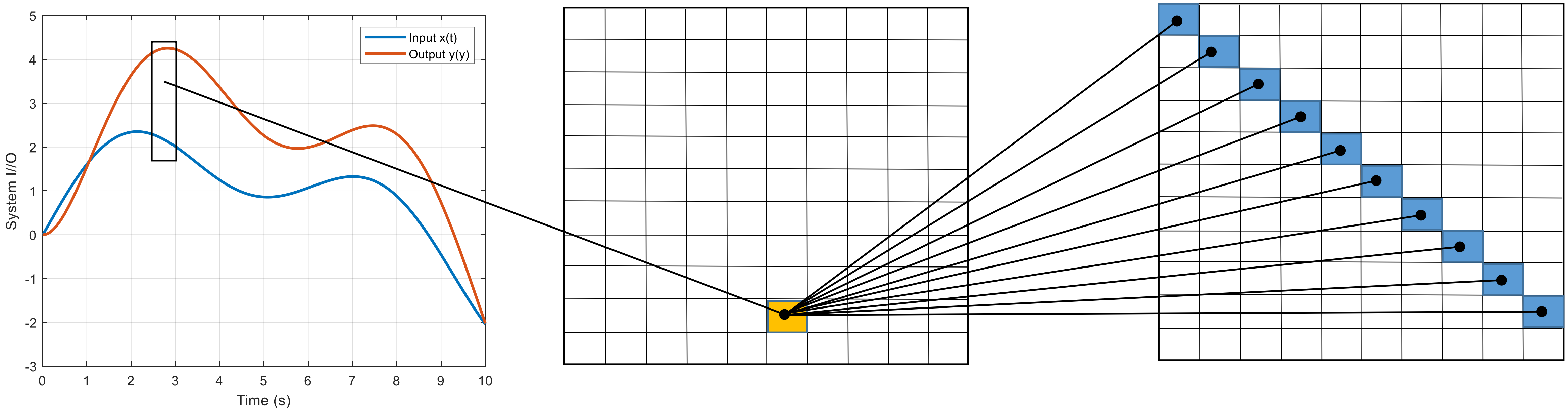
Conclusions

- Hough Transform successfully applied to system identification
- Hough Transform part of an adaptive controller design
- Pattern recognition for control, system health an fault isolation

Backup

Look at a systems phase plane characteristics

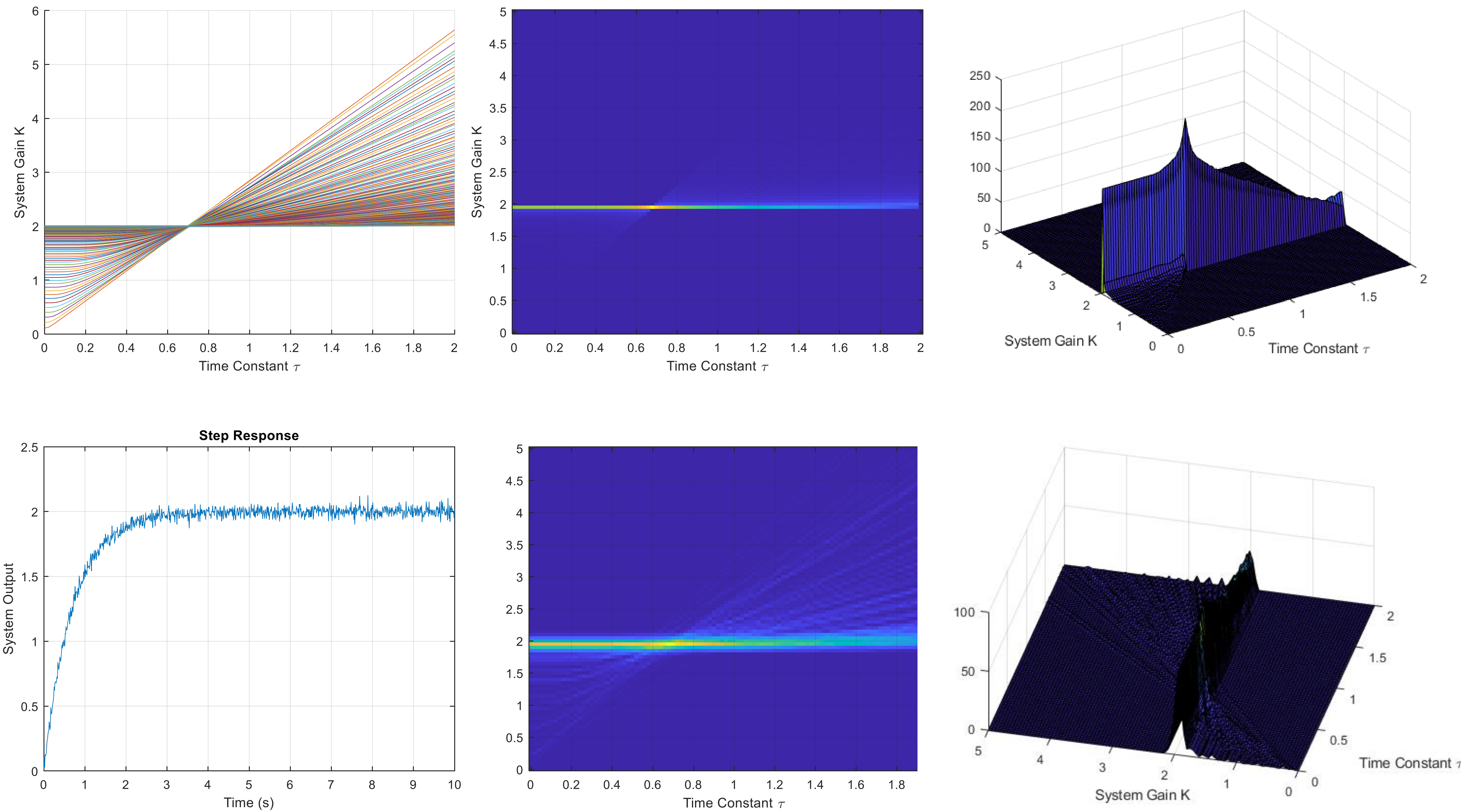
Should work for second order systems



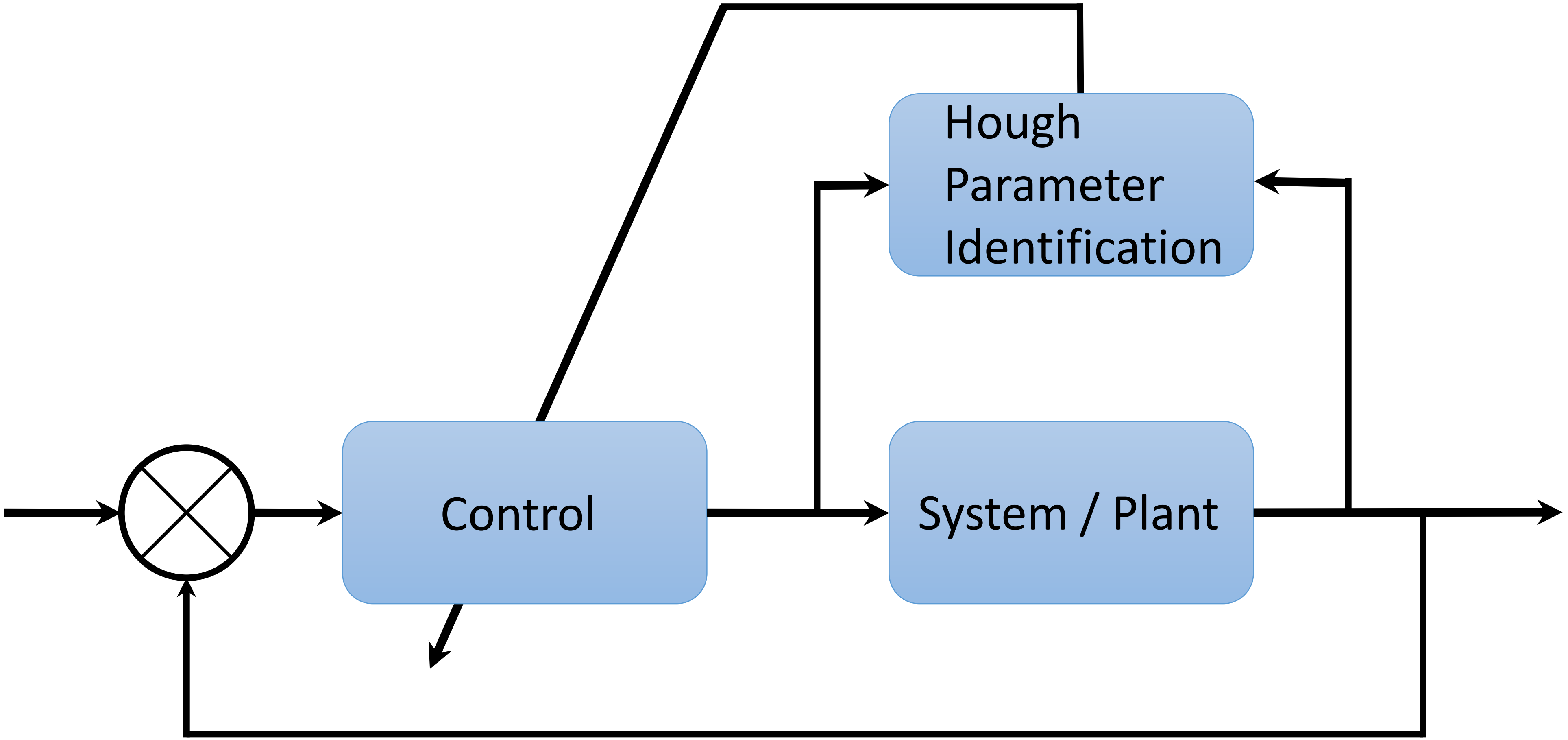
System Response Data

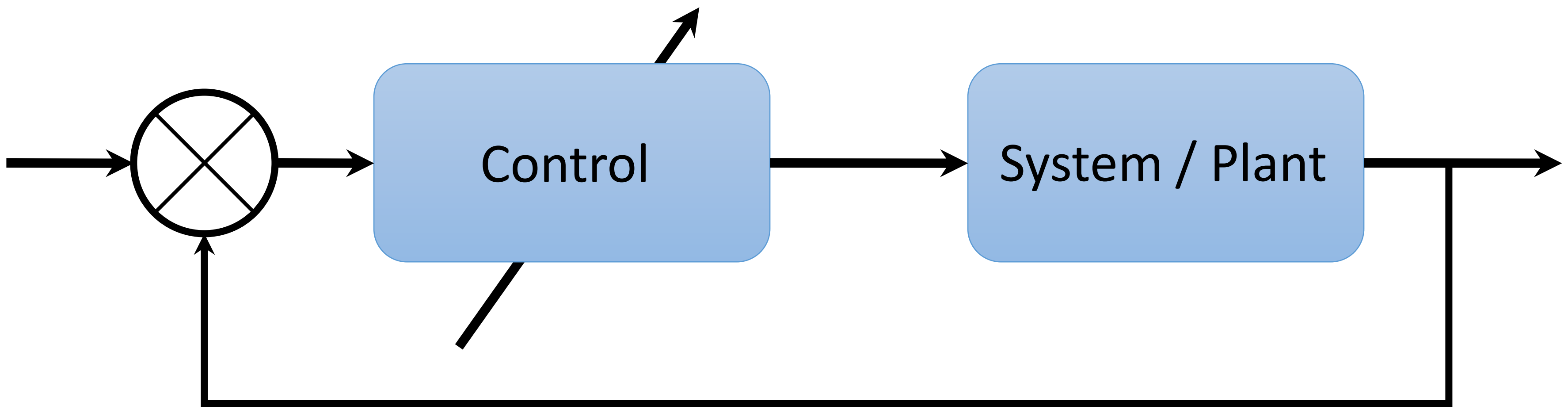
Mapping Response Data to 'pixel' resolution

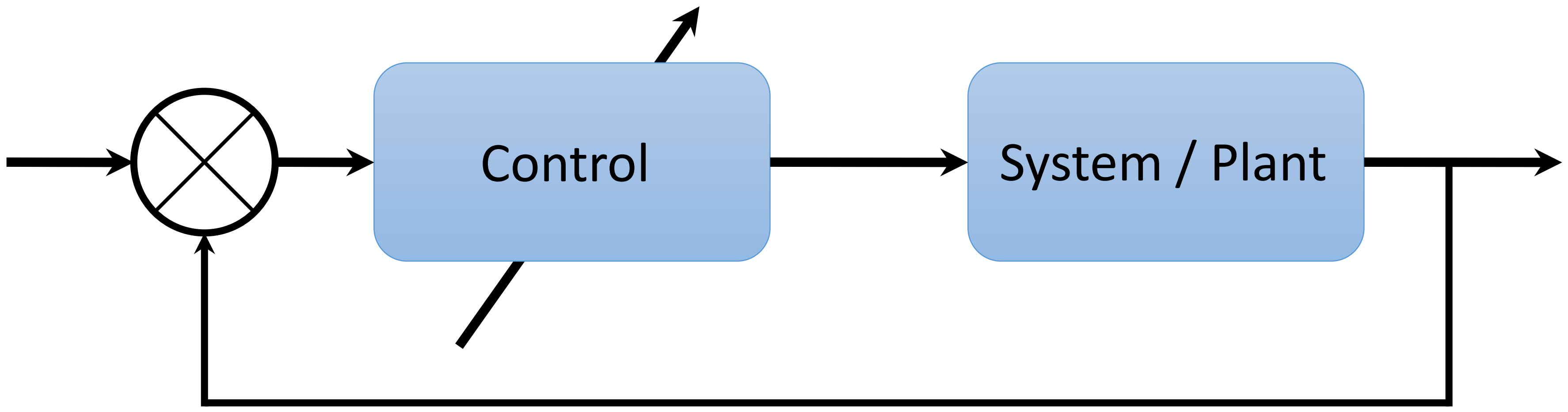
System/Controller Parameter Space


System Gain K
Time Constant τ
System Gain K
Time Constant τ
System Gain K
Time Constant τ
Step Response
System Output
Time (s)
Time Constant τ
System Gain K
Time Constant τ

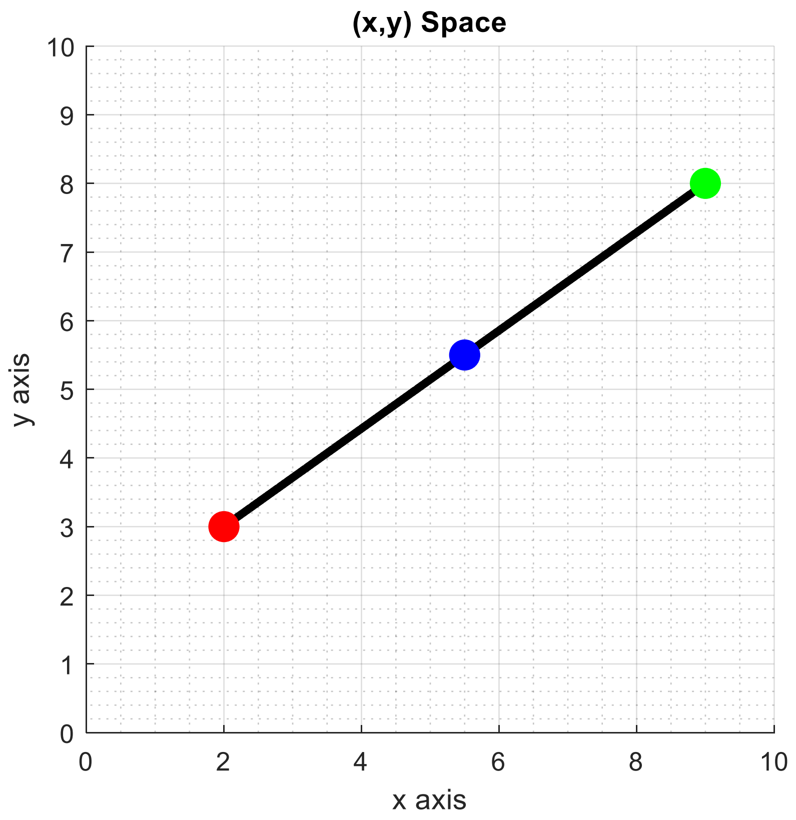
Hough Controller

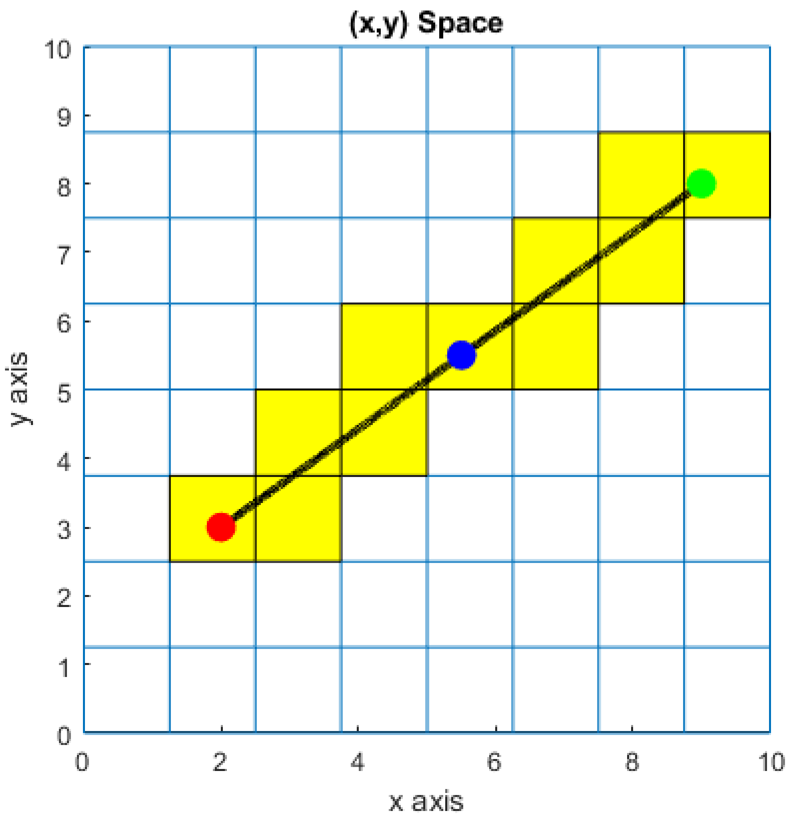


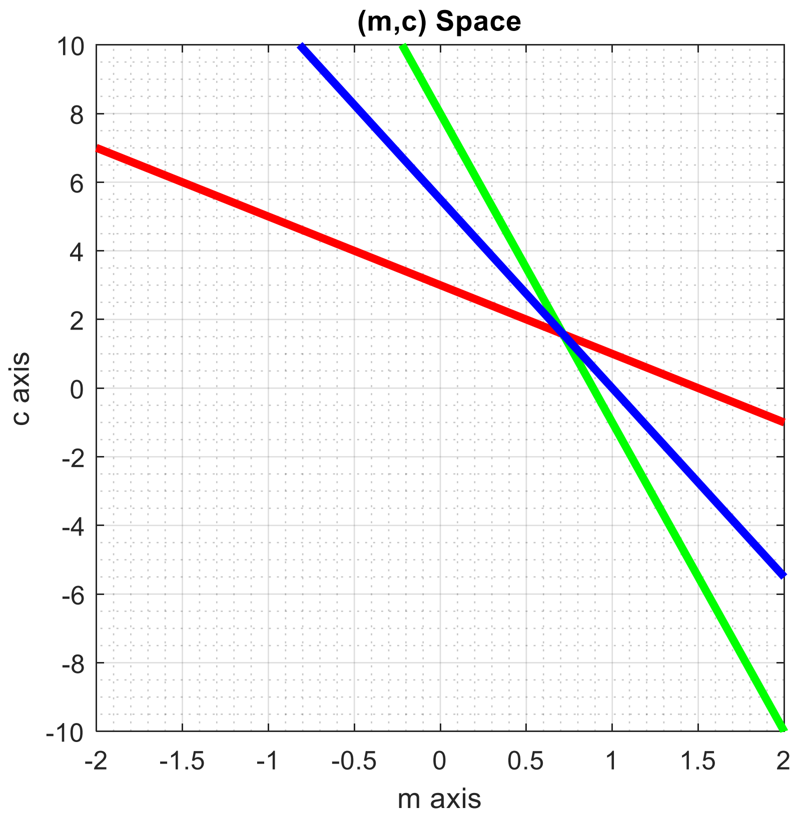

Control
System / Plant


Control
System / Plant

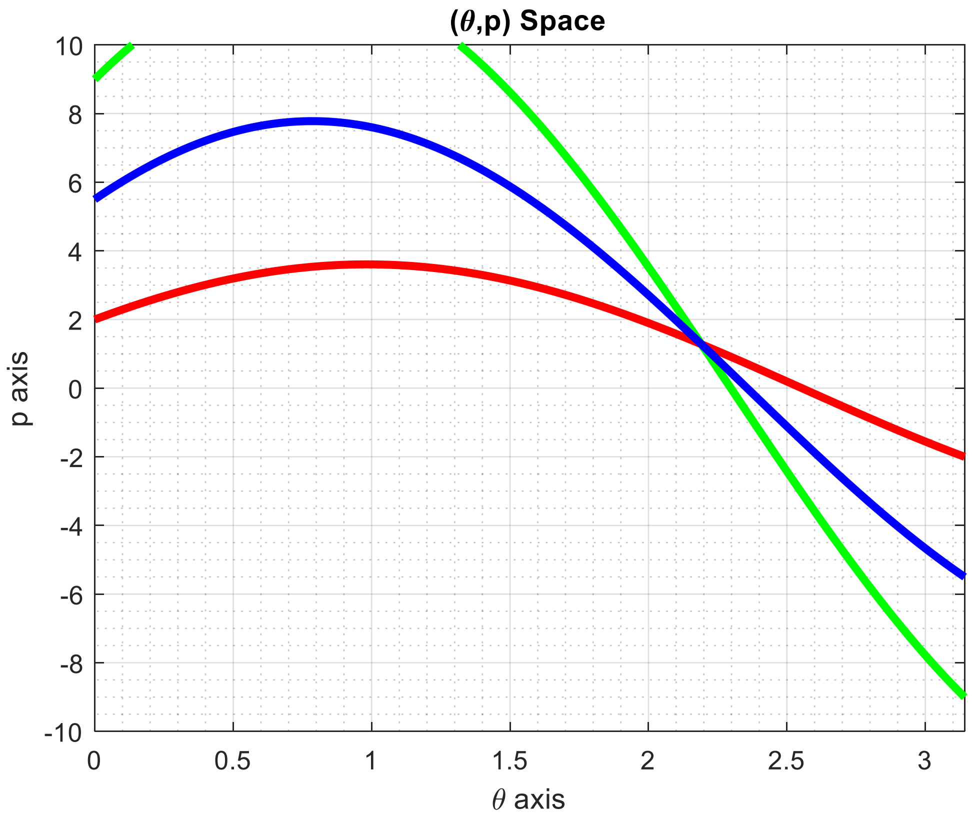
Hough Transform for a straight line







(m,c) Space
c axis
m axis
10
8
6
4
2
0
-2
-4
-6
-8
-10
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2



(θ,p) Space
p axis
θ axis
10
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.5
1
1.5
2
2.5
3

Application to higher order systems (step response)

Consider the system

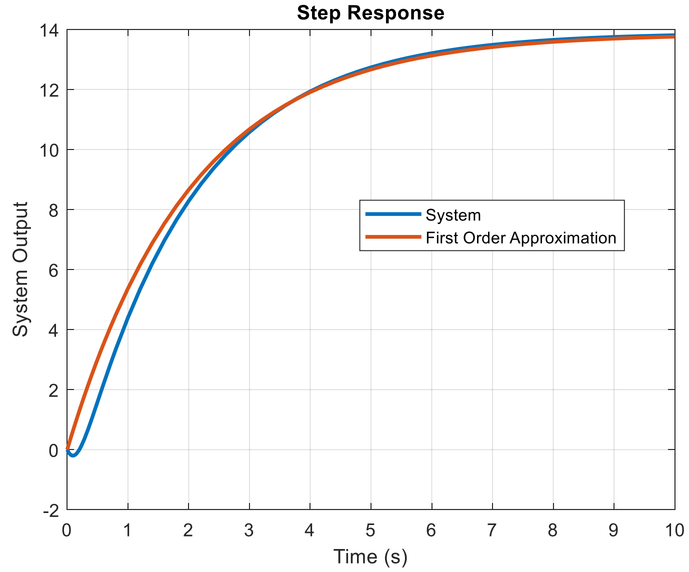
$$G(s) = \frac{-4.283s^2 - 43.32s + 677.9}{s^3 + 20.09s^2 + 103s + 48.81}$$

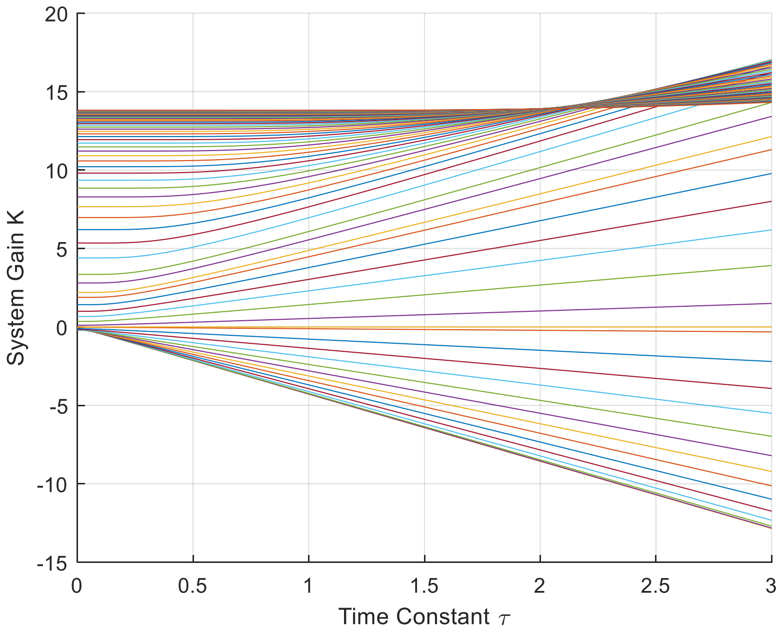
Approach with a first order system identification

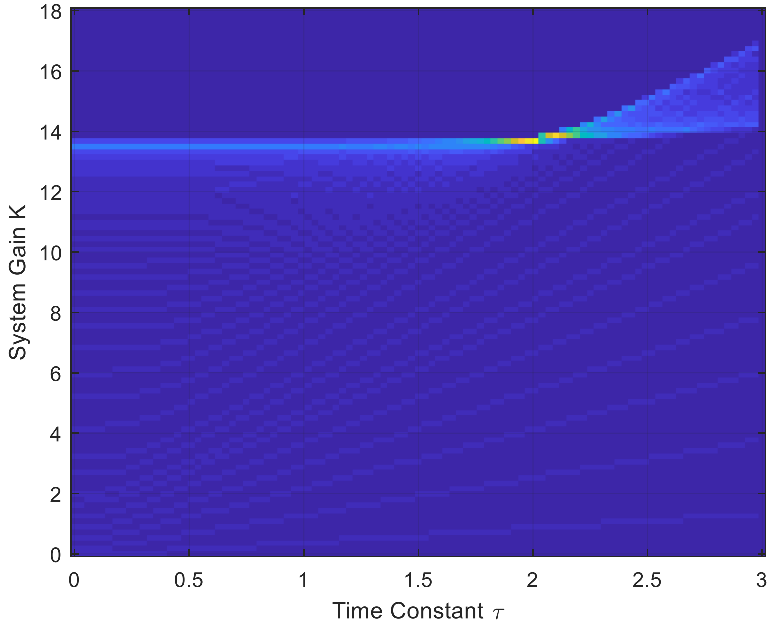
$$G(s) = \frac{13.86}{2.04s + 1}$$

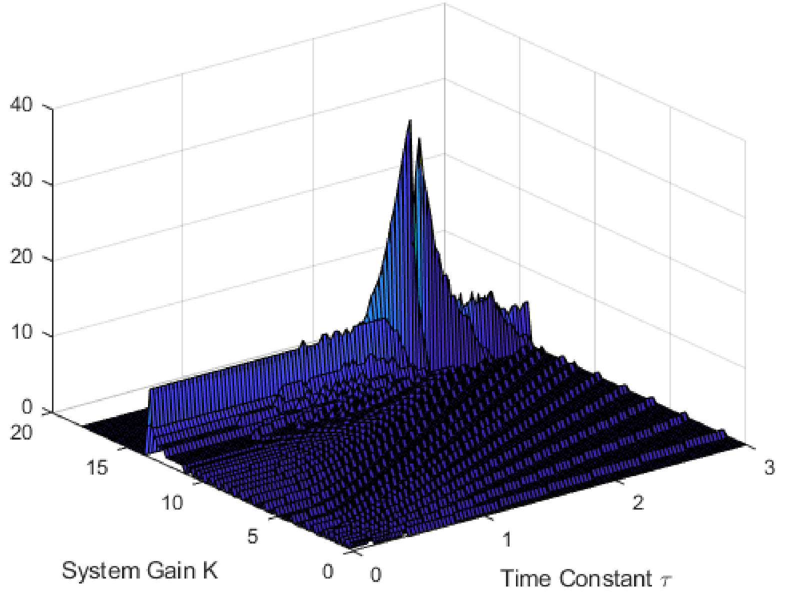
Dominant system pole is -0.53

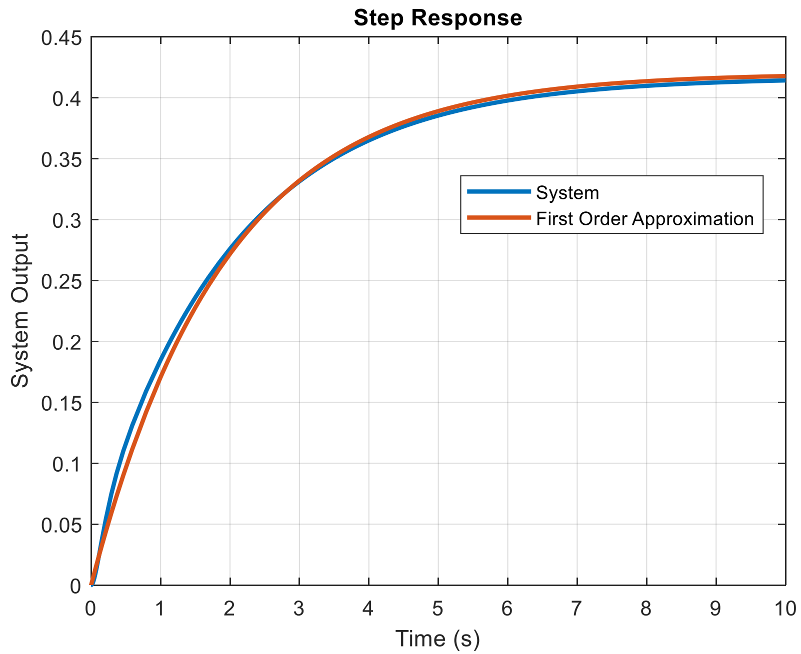
First order approximation is -0.49



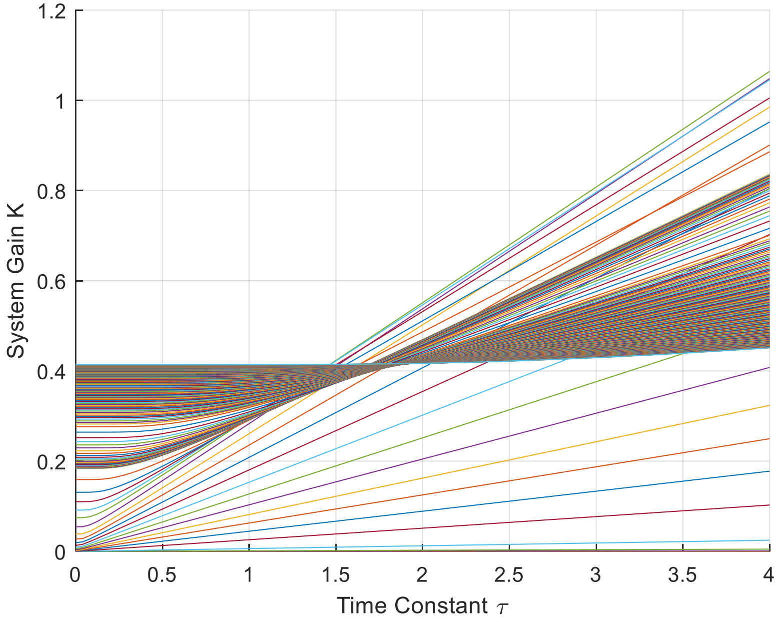




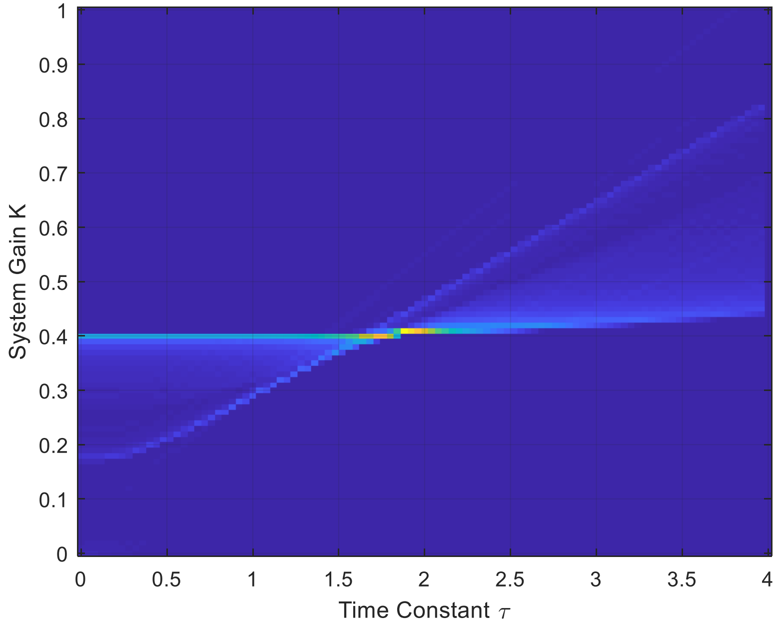



Step Response
System
First Order Approximation
System Output
Time (s)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
1
2
3
4
5
6
7
8
9
10

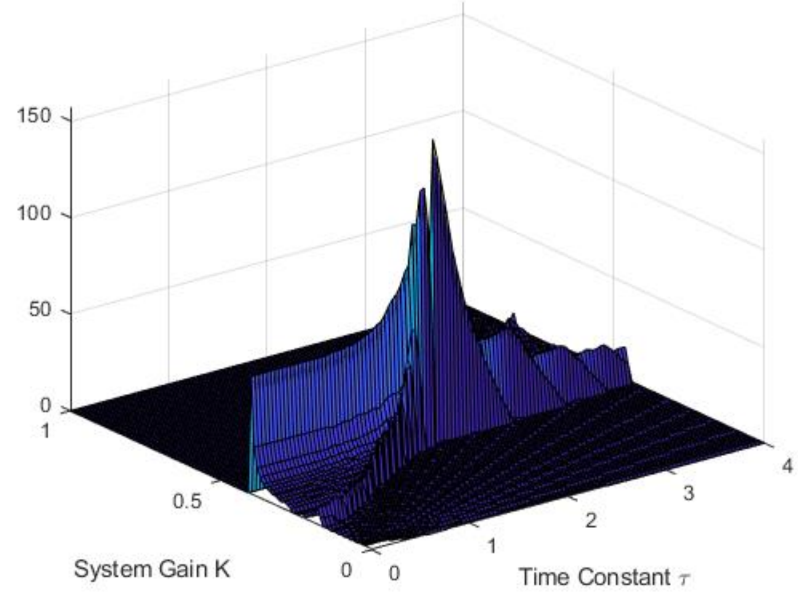
(a)


System Gain K
Time Constant τ
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
3.5
4

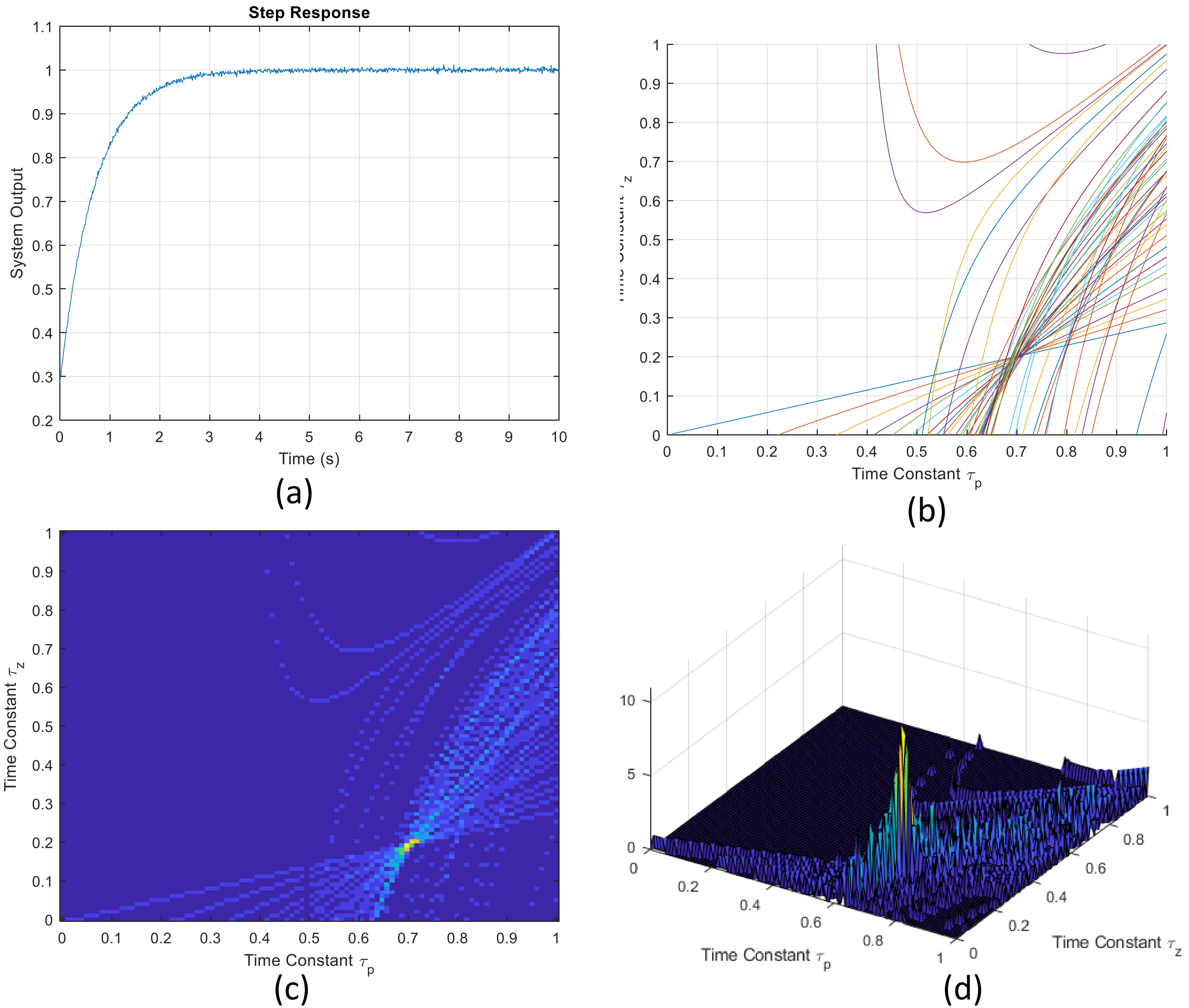
(b)

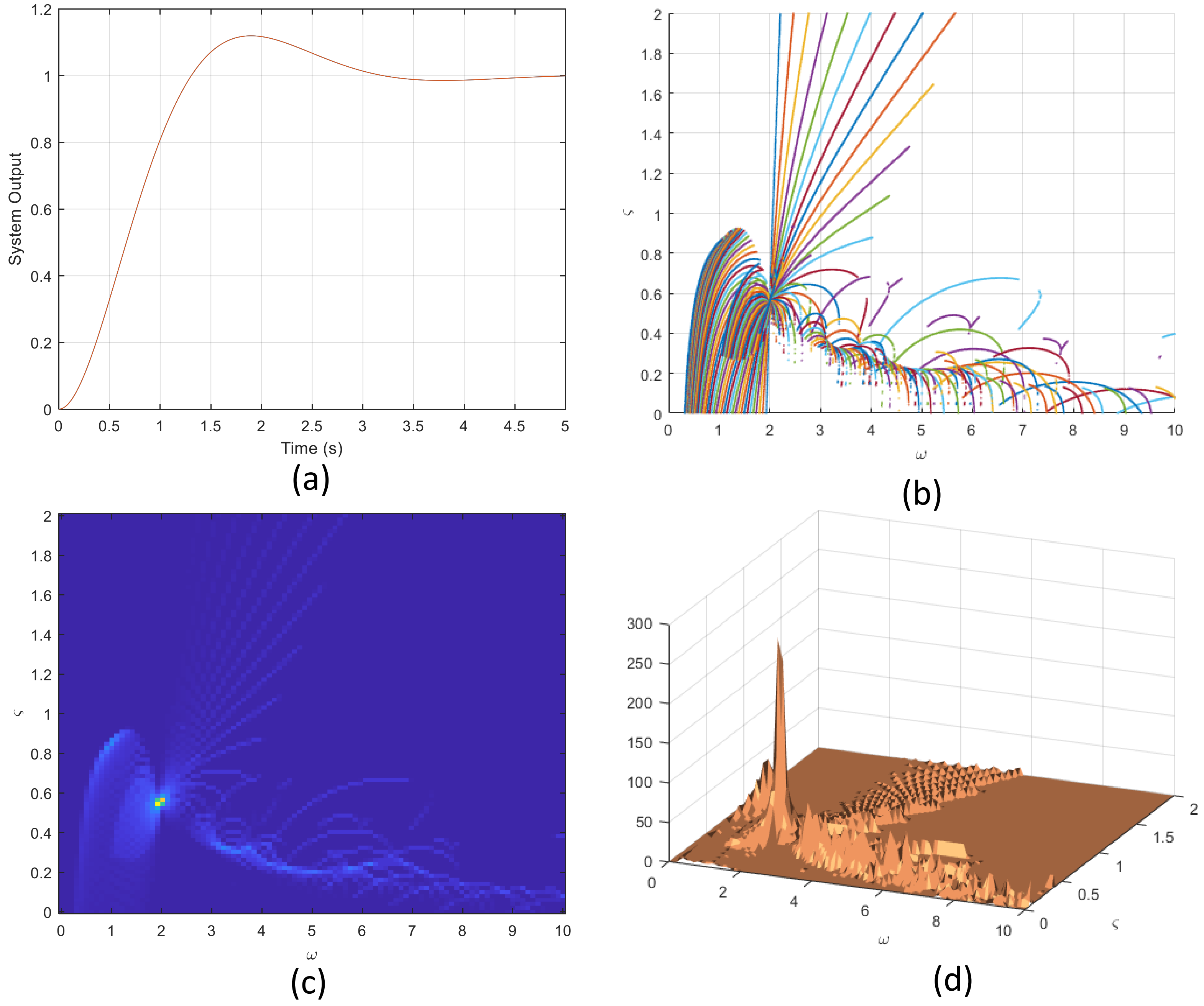

System Gain K
Time Constant τ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4

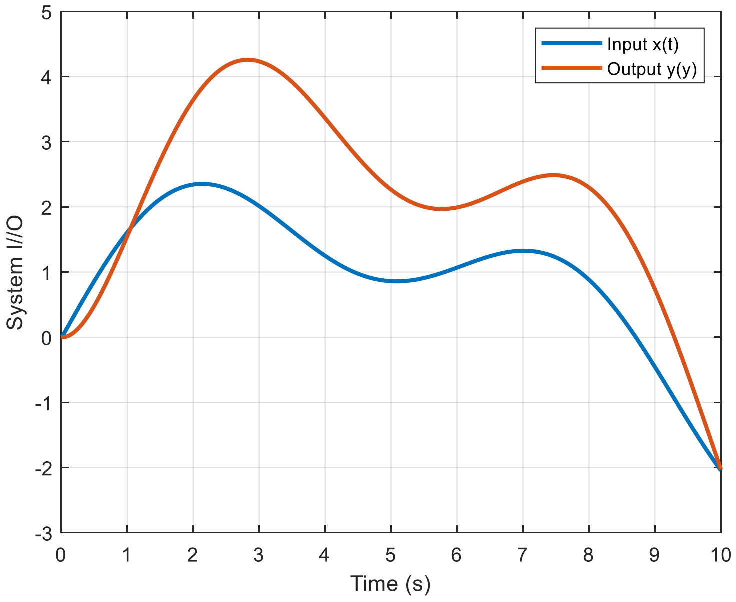
(c)

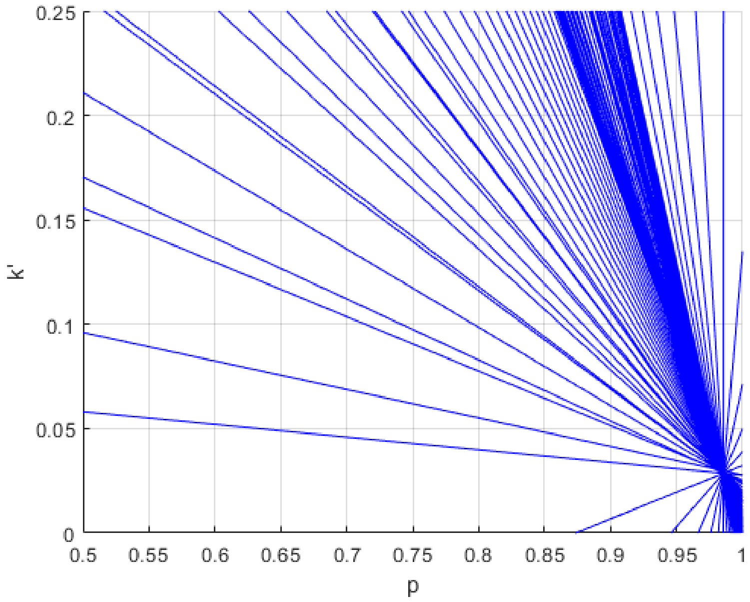

150
100
50
0
1
0.5
0
System Gain K
0
1
2
3
4
Time Constant τ

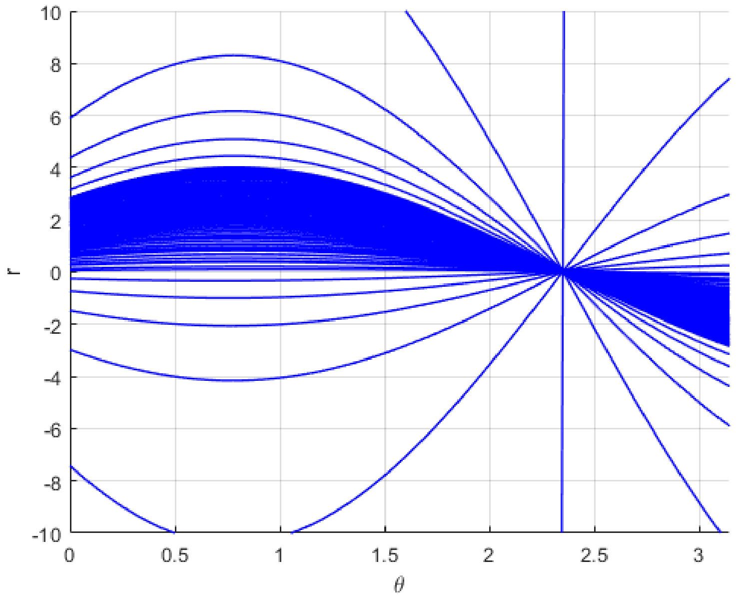
(d)


Step Response
System Output
Time (s)
(a)
Time Constant τ_p
(b)
Time Constant τ_z
Time Constant τ_p
(c)
Time Constant τ_p
Time Constant τ_z
(d)


System Output
Time (s)
(a)
ω
ς
(b)
ω
ς
(c)
ω
ς
(d)


Input x(t)
Output y(y)
System I//O
Time (s)
5
4
3
2
1
0
-1
-2
-3
0 1 2 3 4 5 6 7 8 9 10


k'
p
0.25
0.2
0.15
0.1
0.05
0
0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1


r
θ
10
8
6
4
2
0
-2
-4
-6
-8
-10
0 0.5 1 1.5 2 2.5 3